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THE HOMOLOGY COVERING OF A RIEMANN SURFACE

Dedicated to Tadashi Kuroda on his sixtieth birthday

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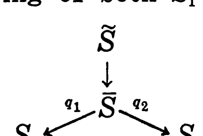
The Riemann surface $\hat{S}$ is an Abelian cover of S if there is a regular covering $p: \hat{S} \to S$ where the group of deck transformations is Abelian.

The *homology covering* $p: \tilde{S} \to S$ is the highest Abelian covering of S; i.e., it is the covering corresponding to the commutator subgroup of $\pi_1(S)$.

THEOREM. *Let S_1 and S_2 be closed Riemann surfaces, where S_m has genus $g_m \geqq 2$. Suppose S_1 and S_2 have conformally equivalent homology covering surfaces. Then S_1 and S_2 are conformally equivalent.*

PROOF. We regard S_1 and S_2 as having the same homology cover $\tilde{S}$. Let Γ_m be the group of deck transformations for $\tilde{S}$ covering S_m; i.e., $\tilde{S}/\Gamma_m = S_m$. Let Γ be the group of conformal self-maps of $\tilde{S}$ generated by Γ_1 and Γ_2. It is well known that $\tilde{S}$ is a surface of infinite genus, so the full group of conformal automorphisms of $\tilde{S}$ is discontinuous; in particular, Γ acts discontinuously. Set $S = \tilde{S}/\Gamma$. Note that while Γ_1 and Γ_2 both act freely, Γ need not.

Since S_1 and S_2 are compact, they are finite sheeted (possibly branched) coverings of S; i.e., both Γ_1 and Γ_2 are of finite index in Γ. It then follows that $\bar{\Gamma} = \Gamma_1 \cap \Gamma_2$ is of finite index in both Γ_1 and Γ_2. Let $\bar{S} = \tilde{S}/(\Gamma_1 \cap \Gamma_2)$. We have the following diagram of coverings (note that $\bar{S}$ is a smooth (unbranched) covering of both S_1 and S_2):



Since the defining subgroup of the covering $q_m: \bar{S} \to S_m$ contains the commutator subgroup, it is normal, and the group of deck transformations is Abelian; i.e., $q_m: \bar{S} \to S_m$ is a regular Abelian covering.

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