

## LINES OF PRINCIPAL CURVATURE FOR MAPPINGS WITH WHITNEY UMBRELLA SINGULARITIES

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**Abstract.** The pattern of lines of principal curvature near Whitney umbrella singularities of mappings of surfaces  $M$  into  $R^3$  are established. Sufficient conditions for the stability of the whole configuration of lines of principal curvature, under small perturbations of the mappings are given. These sufficient conditions are satisfied by a dense set in the space of  $C^r$ -mappings of  $M$  into  $R^3$ ,  $r \geq 4$ , endowed with the  $C^2$ -topology.

**1. Introduction.** Let  $M$  be a compact, oriented smooth two-manifold. This work will be concerned with the study of the different stable patterns through which  $M$  can be bended when smoothly mapped into  $R^3$ . The bending pattern of a mapping  $\alpha: M \rightarrow R^3$  will be represented here by the singular points,  $\mathcal{C}_\alpha$ , at which the bending is infinite; the umbilical points  $\mathcal{U}_\alpha$  at which the bending is finite but equal in all directions and; by the family of lines of principal curvature  $\mathcal{F}_\alpha, \mathcal{f}_\alpha$ , defined on  $M - (\mathcal{U}_\alpha \cup \mathcal{C}_\alpha)$ , which represent the directions along which the bending is extremal: maximal along  $\mathcal{F}_\alpha$  and minimal along  $\mathcal{f}_\alpha$ .

These four objects will be assembled together into the principal configuration of the mapping denoted by  $\mathcal{P}_\alpha = (\mathcal{C}_\alpha, \mathcal{U}_\alpha, \mathcal{F}_\alpha, \mathcal{f}_\alpha)$ .

The study of principal configurations on surfaces goes back to the classical works of Dupin, Darboux and Cayley. The reader is referred to the Introduction of Gutierrez—Sotomayor [G-S. 1] for a discussion and bibliographical references concerning the general background for the study of principal configurations of immersions, i.e., for mappings  $\alpha$  with  $\mathcal{C}_\alpha$  empty.

This paper will be mainly concerned with the study of the global features of principal configurations  $\mathcal{P}_\alpha$  that remain undisturbed under small perturbations of the mapping  $\alpha$ . The case of immersions was studied in [G-S. 1], [G-S. 2] under the name of Principal Structural Stability.

In this work (Theorem 1) the principal configurations near Whitney umbrella singularities [Wh] of mappings of surfaces  $M$  into  $R^3$  will be established. The local conclusions obtained here combined with the conditions given in [G-S. 1] for the case of non-singular mappings (immersions), provide sufficient conditions for the stability of the whole configuration