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QUATERNIONIC SUBMANIFOLDS IN QUATERNIONIC SYMMETRIC SPACES

HIROYUKI TASAKI

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1. Introduction. The purposes of this paper are to classify quaternionic submanifolds in a quaternionic symmetric space and to investigate the homology classes represented by quaternionic submanifolds in a compact quaternionic symmetric spaces. The following results motivate the subject of this paper. Quaternionic submanifolds in a quaternionic Kähler manifold are minimal stable submanifolds and compact ones are homologically volume minimizing, as were proved by the author in [13].

Here we shall give the definitions of quaternionic Kähler manifolds and quaternionic submanifolds and state some properties of quaternionic submanifolds. A $4n$-dimensional connected Riemannian manifold M is called a *quaternionic Kähler manifold*, if M has the following property: There is a point x in M such that, through an identification of $T_x(M)$ with $\boldsymbol{H}^n$, the linear holonomy group of M at x is contained in $Sp(n)Sp(1)$. In this situation, take a piecewise smooth curve τ from x to y for any point y in M and put

$$S_y = P_\tau Sp(1) P_\tau^{-1} ,$$

where P_τ is the parallel translation along the curve τ. S_y is independent of the choice of τ, because $Sp(1)$ is a normal subgroup of $Sp(n)Sp(1)$. We call $S = \{S_y\}_{y \in M}$ a *quaternionic structure* on M. A connected submanifold N of M is calld a *quaternionic submanifold* in M, if $T_y(N)$ is invariant under the action of S_y for each y in N. Alekseevskii [1] proved that a