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SEMI-SYMMETRIC LORENTZIAN HYPERSURFACES

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Dedicated to Professor Dr. A. Lichnerowicz for his seventieth birthday

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1. Introduction. Nomizu [2] classified semi-symmetric hypersurfaces in Euclidean spaces. In this paper, we shall give a classification of semi-symmetric *Lorentzian hypersurfaces* in Minkowski spaces. We recall that a semi- or pseudo-Riemannian manifold M is said to be *semi-symmetric*, if it satisfies the condition $R \cdot R = 0$, whereby R is the Riemmann-Christoffel curvature tensor of M and where the first tensor acts on the second one as a derivation. Semi-symmetry is a proper generalization of local symmetry, and was first studied by Cartan and Lichnerowicz. Recently, a general study of semi-symmetric Riemannian manifolds was made by Szabó [4].

The main results of this paper can be stated as follows.

THEOREM 1. *Let M^n be a Lorentzian hypersurface of dimension n in a Minkowski space $\boldsymbol{R}_1^{n+1}$. Suppose that the type number $k(x)$ is $\geqq 3$ at a point x of M^n. Then M^n is semi-symmetric at x if and only if the shape operator A_x of M^n at x has the form*

$$(1) \qquad A_x = \left[\begin{array}{c|c} \lambda I_{k(x)} & 0 \\ \hline 0 & 0_{n-k(x)} \end{array}\right], \quad \lambda \in \boldsymbol{R}\setminus\{0\}$$

with respect to a suitable orthonormal frame of T_xM^n.

THEOREM 2. *Let M^n be a connected and complete Lorentzian hypersurface of dimension n in a Minkowski space $\boldsymbol{R}_1^{n+1}$. Suppose that the type number is $\geqq 3$ at least at one point of M^n. Then M^n is semi-symmetric if and only if*

(a) $M^n = S_1^k \times \boldsymbol{R}^{n-k}$

or

(b) $M^n = S^k \times \boldsymbol{R}_1^{n-k}$,

for some $k \geqq 3$. In case (a), *S_1^k is a Lorentzian hypersphere in a Minkowski subspace $\boldsymbol{R}_1^{k+1}$ of $\boldsymbol{R}_1^{n+1}$ and $\boldsymbol{R}^{n-k}$ is a Euclidean subspace of $\boldsymbol{R}_1^{n+1}$ orthogonal to $\boldsymbol{R}_1^{k+1}$. In case* (b), *S^k is a hypersphere in a Euclidean*