Tôhoku Math. Journ.
39 (1987), 71–79.

ALMOST PERIODIC SOLUTIONS OF A SYSTEM OF INTEGRODIFFERENTIAL EQUATIONS

SATORU MURAKAMI

(Received January 25, 1986)

The purpose of this article is to discuss the existence of almost periodic solutions of a system of almost periodic integrodifferential equations

$$(E)\qquad \dot{x}_i(t) = h_i(x_i(t))\Big\{b_i(t) - a_{ii}(t)x_i(t) - \sum_{\substack{j=1\\ j\neq i}}^{k} a_{ij}(t)\int_{-\infty}^{t} K_{ij}(t-u)G_i(x_j(u))du\Big\}\,,\quad i = 1, 2, \cdots, k\,,$$

which describes a model of the dynamics of a k-species system in mathematical ecology when $h_i(s) = G_i(s) \equiv s$. When $h_i(s) = G_i(s) \equiv s$ and $a_{ij}(t)$, $b_i(t)$ are ω-periodic, Gopalsamy [2] has recently discussed the existence of ω-periodic solutions of System (E) under some conditions. In order to obtain an ω-periodic solution of System (E), he has investigated the existence of ω-periodic solutions of another system

$$(E_0)\qquad \dot{x}_i(t) = h_i(x_i(t))\Big\{b_i(t) - a_{ii}(t)x_i(t) - \sum_{\substack{j=1\\ j\neq i}}^{k} a_{ij}(t)\int_{t-\omega}^{t} \sum_{r=0}^{\infty} K_{ij}(t-u+r\omega)G_i(x_j(u))du\Big\}\,,\quad i = 1, 2, \cdots, k\,,$$

instead of the original system (E), because any ω-periodic solution of System (E) is also an ω-periodic solution of System (E_0) and vice versa. As easily seen, however, we cannot directly employ Gopalsamy's idea when System (E) is almost periodic. In this article, we shall investigate some stability properties of a solution of System (E), and consequently obtain an almost periodic solution of System (E). We emphasize that our result contains Theorem 2.1 in [2] as a special case.

In what follows, we denote by R^k the k-dimensional real Euclidean space and by $|x|$ the norm of $x \in R^k$. Throughout this paper, we suppose that the functions h_i, b_i, a_{ij}, K_{ij} and G_i in System (E) are real-valued continuous functions on $R := R^1$ and that the following conditions are satisfied: