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ON A METHOD TO CONSTRUCT ANALYTIC ACTIONS OF NON-COMPACT LIE GROUPS ON A SPHERE

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0. Introduction. Let M be a square matrix of degree n with real coefficients, that is, $M \in M_n(\boldsymbol{R})$. We say that M satisfies the *outward transversality condition* if

$$\frac{d}{dt}\|\exp(tM)x\| > 0 \quad \text{for each} \quad x \in \boldsymbol{R}_0^n = \boldsymbol{R}^n - \{0\} \quad \text{and} \quad t \in \boldsymbol{R} .$$

In this case, there exists a unique real valued analytic function τ on $\boldsymbol{R}_0^n$ such that $\|\exp(\tau(x)M)x\| = 1$, and hence we can define an analytic mapping π^M of $\boldsymbol{R}_0^n$ onto the unit $(n-1)$-sphere S^{n-1} by $\pi^M(x) = \exp(\tau(x)M)x$.

Let G be a Lie group, $\rho\colon G \to GL(n, \boldsymbol{R})$ a matricial representation, and M a square matrix of degree n with real coefficients satisfying the outward transversality condition. We can define an analytic mapping ξ: $G \times S^{n-1} \to S^{n-1}$ by $\xi(g, x) = \pi^M(\rho(g)x)$, and we see that ξ is an analytic G-action on S^{n-1} if $\rho(g)M = M\rho(g)$ for any $g \in G$. We call ξ a *twisted linear action* of G on S^{n-1} associated to the representation ρ. In particular, if M is the identity matrix, we call ξ a *linear action* of G on S^{n-1} associated to the representation ρ.

Let G be a compact Lie group and $\rho\colon G \to GL(n, \boldsymbol{R})$ a matricial representation. Then we shall show that any twisted linear action of G on S^{n-1} associated to ρ is equivariantly analytically diffeomorphic to the linear action of G on S^{n-1} associated to ρ. On the other hand, if G is a non-compact Lie group, sometimes we can construct uncountably many topologically distinct twisted linear actions of G associated to only one matricial representation (cf. [4, §6]). We shall study such an example in the final section.

1. Outward transversality condition.

1.1. Let $u = (u_i)$ and $v = (v_i)$ be vectors in $\boldsymbol{R}^n$. As usual, we denote their inner product by $u \cdot v = \sum_i u_i v_i$ and the length of u by $\|u\| = \sqrt{u \cdot u}$.

LEMMA 1.1. *Let $M \in M_n(\boldsymbol{R})$ and assume that M satisfies the outward transversality condition. Then,* (i)