Tôhoku Math. Journ.
39 (1987), 27–40.

A CHARACTERIZATION OF THE CARTAN HYPERSURFACE IN A SPHERE

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(Received January 20, 1986)

Introduction. Let $S^n(c)$ be an n-dimensional sphere of constant curvature c in an $(n+1)$-dimensional Euclidean space $\boldsymbol{R}^{n+1}$. A hypersurface in $S^4 = S^4(1)$ defined by the equation

$$2x_5^3 + 3(x_1^2 + x_2^2)x_5 - 6(x_3^2 + x_4^2)x_5 + 3\times 3^{1/2}(x_1^2 - x_2^2)x_4 + 6\times 3^{1/2}x_1x_2x_3 = 2$$

was investigated by E. Cartan [1], who proved that this space is a homogeneous Riemannian manifold $SO(3)/(\boldsymbol{Z}_2\times\boldsymbol{Z}_2)$ and that the principal curvatures of the hypersurface are equal to $3^{1/2}$, 0 and $-3^{1/2}$ everywhere. This is called the *Cartan hypersurface* in S^4, which is one of isoparametric hypersurfaces in a sphere. The classification of isoparametric hypersurfaces in a real space form has been studied by Münzner [8], Ozeki and Takeuchi [9], Takagi and Takahashi [15] and so on.

For the Cartan hypersurface in a sphere S^4, there seem to be two studies from different points of view: n-dimensional homogeneous hypersurfaces in a real space form were investigated by Kobayashi [5] and Takahashi [16], who gave the classification except when $n = 3$ and type number 2, or when $n = 2$. Takagi [14] noted that the exceptional case actually characterizes the Cartan hypersurface, i.e., the 3-dimensional Cartan hypersurface is the only connected homogeneous hypersurface in S^4 whose type number is equal to 2 at some point. The other is due to Peng and Terng [10], who investigated closed minimal hypersurfaces in a sphere the square length of whose second fundamental form is constant, thereby characterized the Cartan hypersurface in S^4.

The purpose of this paper is to give another characterization of the Cartan hypersurface in a sphere from the standpoint of Ricci tensor. In §1, we recall briefly the theory of hypersurfaces in a real space form and in §2 we outline some properties of isoparametric hypersurfaces, one of which is called a Cartan hypersurface. The Ricci tensor S with components R_{ij} is said to be *cyclic-parallel*, if the cyclic sum of the covariant

* Partially supported by JSPS and KOSEF.
** Partially supported by the Grants-in-Aid for Scientific Research, The Ministry of Education, Science and Culture, Japan