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NEGATIVITY OF THE CURVATURE OPERATOR OF A BOUNDED DOMAIN

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

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Introduction. Let D be a bounded domain in $\boldsymbol{C}^n$ equipped with the Bergman metric g. Let $R_{a\bar{b}c\bar{d}}$ be the components of the Riemannian curvature tensor of g. The curvature operator Q of g at a point $p \in D$ is, by definition, the endomorphism

$$\xi_{ab} \mapsto R_a{}^{cd}{}_b(p)\xi_{cd}$$

of the 2-symmetric tensor product of the holomorphic tangent space at p. The eigenvalues of Q are holomorphically invariant and are all real because Q is self-adjoint with respect to the Hermitian inner product induced from g. In particular, if D is homogeneous, then the eigenvalues of Q do not depend on the point of D under consideration. The following is well-known ([4], [5]): If D is irreducible symmetric and the operator Q is negative definite, then D is holomorphically equivalent to a ball. Concerning this, we consider the following two problems:

(A) Let D be a, not necessarily irreducible, homogeneous domain in $\boldsymbol{C}^n$. Suppose Q is negative definite. Then is D holomorphically equivalent to a ball?

(B) Does there exist a bounded domain which is not holomorphically equivalent to a ball and for which Q is negative definite?

Our aim of the present note is to show that problem (A) has an affirmative answer by means of the theory of normal j-algebras by Pyatetskii-Shapiro [8] (Theorem 1), and to show that a Thullen domain, which is holomorphically inequivalent to a ball, has negative definite curvature operator (Proposition 4). Problem (B) is also affirmative in view of the deformation theory by Greene and Krantz [6], [7].

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1. Homogeneous bounded domains. In this section we shall show the following.