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HYPERCUSPIDALITY OF AUTOMORPHIC CUSPIDAL REPRESENTATIONS OF THE UNITARY GROUP $U(2, 2)$

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Introduction. In this paper, we study the hypercuspidality of automorphic cuspidal representations of the unitary group $U(2, 2)$.

The hypercuspidality in the case of the symplectic group was introduced by Piatetski-Shapiro [6]. For $G = GSp_4$, a cusp form f on G_A is called hypercuspidal if the Whittaker function corresponding to f vanishes (cf. [7]).

Analogously, we define the hypercuspidality in the case of $U(2, 2)$ by the vanishing of some Whittaker functions occuring in the Fourier expansion of the cusp form. More precisely, for a cusp form f on $U(2, 2)$, we consider the Fourier expansion of f with respect to the center of the unipotent radical of the Borel subgroup. Then we obtain two Whittaker functions W_f and V_f, where W_f is the ordinary Whittaker function and V_f is as defined in Section 1. We note that in the case of Sp_4, the function V_f did not appear in a similar Fourier expansion of a cusp form f. In terms of these functions, we say f is U-cuspidal (resp. N-cuspidal) if W_f (resp. V_f) vanishes. Further, if both of the functions W_f and V_f vanish, f is called hypercuspidal.

Next, using the notion of the dual reductive pair, we investigate cuspidal representations obtained from the Weil-lifting of cuspidal representations of $U(1, 1)$ or $U(2, 1)$. Symbolicically, $U(1, 1)$, $U(2, 1)$, $\cdots$, denote unitary groups over a global field of degree 2, 3, $\cdots$, with maximal index. Let τ be a cuspidal representation of $U(1, 1)$ or $U(2, 1)$ and $\Theta(\tau, \psi)$ a cuspidal representation of $U(2, 2)$ obtained from the Weil-lifting of τ. For $\varphi \in \tau$, let f_φ be an element in $\Theta(\tau, \psi)$ corresponding to φ. By an explicit computation of the Fourier coefficients of f_φ, we have relations between Whittaker functions of φ and f_φ (Lemma (3.2), Theorem (4.3) and Proposition (4.4)). Using these relations, we prove the non-vanishing of $\Theta(\tau, \psi)$. Further, under an additional assumption, we obtain some results about the hypercuspidality of $\Theta(\tau, \varphi)$ (Theorem (3.1) and Corollary to Proposition (4.4)).

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