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INFINITESIMAL DEFORMATIONS OF GENERALIZED CUSP SINGULARITIES

SHOETSU OGATA

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0. Introduction. In [Hz], Hirzebruch studied Hilbert modular surfaces which are the compactifications of $\boldsymbol{H}^2/SL_2(\mathcal{O})$ determined by addition of a finite number of points called "cusps", where $\boldsymbol{H} := \{z \in \boldsymbol{C}; \operatorname{Im} z > 0\}$ is the upper half plane and $\mathcal{O}$ is the ring of integers in a real quadratic field. He also constructed the minimal models of these surfaces by using the method of toroidal embeddings [TE]. This method is local, that is, this is performed only near each cusp. Tsuchihashi constructed in [T1] normal isolated singularities, sometimes called "Tsuchihashi cusps", analogous to Hilbert modular cusp singularities by using toroidal embeddings. A Tsuchihashi cusp singularity (V, p) is of the form $V \setminus \{p\} \cong \mathscr{D}/G$, where $\mathscr{D}$ is a tube domain and G is a subgroup of $\operatorname{Aut}(\mathscr{D})$.

Recall that a tube domain is called a Siegel domain of the *first* kind. We construct in Section 1 a normal isolated singularity (V, p) such that $V \setminus \{p\}$ is isomorphic to a quotient of a Siegel domain of the *second* kind. We would like to call this singularity also a "cusp". It is natural to extend the class of cusp singularities in this way, because the boundary components of the Satake compactification of a quotient of a bounded symmetric domain are also called cusps in a generalized sense.

EXAMPLE. Let F be a totally real algebraic number field of degree ν, F' a totally imaginary quadratic extention of F, B a central division algebra of degree d over F' with an involution of the second kind and $h \in M_\mu(B)$ a Hermitian matrix with Witt index one, i.e., h is conjugate to

$$\left[\begin{array}{cc|c} 0 & 1 & \\ 1 & 0 & 0 \\ \hline 0 & & * \end{array}\right].$$

Set $G_{\boldsymbol{Q}} := R_{F/\boldsymbol{Q}}(SU(h, B/F'/F))$ with Weil's restriction functor $R_{F/\boldsymbol{Q}}$. Then we get

$$G_{\boldsymbol{R}} = \prod_{i=1}^{\nu} SU(p_i, q_i)\ , \quad p_i + q_i = \mu\ , \quad p_i \geqq q_i \geqq d\ .$$

Let K be a maximal compact subgroup of $G_{\boldsymbol{R}}$. When $q_i = d$, we get the