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THE K-ENERGY MAP, ALMOST EINSTEIN KÄHLER METRICS AND AN INEQUALITY OF THE MIYAOKA-YAU TYPE

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The purpose of this paper is to give some addenda to the paper [2], in which we showed the boundedness as well as a consequence of the K-energy map under the assumption of the existence of Einstein Kähler metrics. We here show how these results can be refined. The author would like to thank Professor H. Urakawa for valuable suggestions.

Let us first fix our notation. Throughout this paper X is an n-dimensional compact complex manifold with positive first Chern class $c_1(X) > 0$. Let K be the set of all Kähler forms on X representing $2\pi c_1(X)$. To each $\omega \in K$ we associate its Ricci curvature form γ_ω, scalar curvature σ_ω, Laplacian Δ_ω acting on the space of real-valued C^∞-functions $C^\infty(X)$, and $f_w \in C^\infty(X)$ such that $\gamma_\omega - \omega = \sqrt{-1}\partial\bar{\partial} f_\omega$, which is determined up to a suitably chosen constant.

DEFINITION. For ω_0, $\omega_1 \in K$, we choose a smooth path $\omega_t = \omega_0 + \sqrt{-1}\partial\bar{\partial} u_t \in K$ connecting ω_0 and ω_1, where $u_t \in C^\infty(X)$ and $t \in [0, 1]$. Hereafter when we encounter notation like γ_{ω_t}, σ_{ω_t}, Δ_{ω_t}, f_{ω_t}, ∇_{ω_t}, we simplify their subscripts in the form γ_t, σ_t, Δ_t, f_t, ∇_t. Using this convention, define three functions I, J, M on $K \times K$ as follows:

$$I(\omega_0, \omega_1) = \int_X u_1(\omega_0^n - \omega_1^n)/V ,$$

$$J(\omega_0, \omega_1) = \int_0^1 dt \int_X \frac{du_t}{dt}(\omega_0^n - \omega_t^n)/V ,$$

$$M(\omega_0, \omega_1) = -\int_0^1 dt \int_X \frac{du_t}{dt}(\sigma_t - n)\omega_t^n/V ,$$

where $V = \int_X \omega_0^n$. For ω_0 in K, we define the corresponding K-energy map $\mu = \mu_{\omega_0}$ from K to R by $\mu(\omega) = M(\omega_0, \omega)$ for $\omega \in K$. Set

$$K^+ = \{\omega \in K \mid \gamma_\omega > 0, \text{ i.e. } \gamma_\omega \text{ is positive definite on } X\} .$$

I, J, M are all well-defined and have nice properties (see [1], [7]).

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