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CONTACT HYPERSURFACES OF A COMPLEX HYPERBOLIC SPACE

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0. Introduction. A differentiable manifold is said to be contact if it admits a linear functional f on the tangent bundle satisfying $f\wedge(df)^{n-1}\neq 0$. The investigation of this as an intrinsic condition has received considerable study, (see [1]). As a real hypersurface of a complex space form is almost contact, it is natural to ask: when is a real hypersurface of a complex space form extrinsically contact?

Such investigations have been carried out successfully for real hypersurfaces of complex Euclidean spaces, [6], and of complex projective space, [4], but until now not for real hypersurfaces of complex hyperbolic space. In this study, contact hypersurfaces of a complex hyperbolic space are classified using the congruence results of [7] in terms of the examples constructed in [7]. In brief: complete connected contact hypersurfaces of $CH^n(-4)$, $n\geqq 3$, are shown to be congruent to geodesic hyperspheres, horospheres or tubes of positive radii around totally geodesic n-dimensional real hyperbolic space forms imbedded in $CH^n(-4)$.

Along the way, a related condition, originally investigated in [5], is taken care of similarly: a complete connected real hypersurface of $CH^n(-4)$ whose induced almost contact structure commutes with its second fundamental form is congruent to a horosphere or a tube of radius $r>0$ around a totally geodesic $CH^p(-4)$, $0\leqq p\leqq n-1$.

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1. Real hypersurfaces of $CH^n(-4)$. Let $CH^n(-4)$, $n\geqq 2$, denote a complex hyperbolic space with the Bergman metric tensor, i.e., a complex space form of constant holomorphic sectional curvature -4. Let M^{2n-1} be a real hypersurface of CH^n, ∇ and $\bar{\nabla}$ be the metric connections on M and CH^n, respectively, so that the Gauss and Weingarten formulae can