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THE GALOIS GROUPS OF THE POLYNOMIALS $x^n + ax^s + b$, II

HIROYUKI OSADA

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Introduction. In the previous paper [3], we have shown that the Galois group of a polynomial $f(x) = x^n + ax^s + b$ (with rational integers a and b) over the rational number field $\boldsymbol{Q}$ is isomorphic to the symmetric group S_n of degree n under the following conditions:

(1) $f(x)$ is irreducible over $\boldsymbol{Q}$.

(2) $a = a_0 c^n$, $b = b_0 c^n$ and $(a_0 c(n-s)s, nb_0) = 1$ (relatively prime).

(3) $|D_0(f)|$ is not a square, where

$$D_0(f) = n^n b_0^{n-s} + (-1)^{n-1} s^s (n-s)^{n-s} a_0^n c^{ns}$$

is a factor of the discriminant $D(f)$ of $f(x)$.

(4) $p \| b_0$ for some prime number p.

(5) There exists a prime number q such that $q|s$ and $k < q$ for any positive integer k with $k|n$ and $k < s/2$.

In this paper, we shall first show that the same result holds without the assumption (5) (Theorem 1). Further, we shall show that there exist infinitely many polynomials $x^n + ax^s + p$ satisfying the above conditions (1), (2), (3) and (4) (Theorem 2).

By Hilbert's irreducibility theorem [2], there exist infinitely many Galois extensions with Galois group S_n or A_n for any n. Schur [4, p. 193-194] gave a criterion for the Galois group of a polynomial over $\boldsymbol{Q}$ to be isomorphic to S_n or to A_n. We here give another criterion for the Galois group of a polynomial over $\boldsymbol{Q}$ to be isomorphic to S_n or to A_n (Theorem 3). As another consequence of our results, we can also construct infinitely many polynomials with the Galois groups A_4, A_5 and A_7 (Corollary 3, Corollary 4 to Theorem 3 and Proposition 2). Besides, we give numerical examples of polynomials with Galois group A_7.

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Let $\boldsymbol{Z}$ be the ring of rational integers. Throughout this paper, we shall denote by K, G and $D(f)$ the splitting field, the Galois group and the discriminant of a polynomial $f(x) \in \boldsymbol{Z}[x]$, respectively.

THEOREM 1. *Let* $f(x) = x^n + ax^s + b$ *be a polynomial in* $\boldsymbol{Z}[x]$. *Let* $a = a_0 c^n$ *and* $b = b_0 c^n$. *Then the Galois group* G *is isomorphic to the*