Tôhoku Math. Journ.
39 (1987), 407-427.

ON A CERTAIN CLASS OF HOMOGENEOUS PROJECTIVELY FLAT MANIFOLDS

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(Received August 1, 1986)

Among the manifolds with a flat projective structure the homogeneous ones are particularly interesting. So far a general classification of such manifolds does not seem to exist. So the interest mostly has focussed on special cases. Agaoka [1] studied left-invariant projectively flat structures on Lie groups in some detail. Also he obtained a complete classification in the case where the Lie group action is the same as for an irreducible Riemannian symmetric space. Vinberg [12] developed a theory for projectively homogeneous bounded domains in $\boldsymbol{R}^n$. Here we will obtain rather complete results about another class of homogeneous projectively flat manifolds, which we call bihomogeneous.

A differentiable manifold M is said to be bihomogeneous if there exists a pair of Lie groups G_1 and G_2, each acting transitively on M and transformations from different groups commuting with each other. When M has a geometric structure, we say it is bihomogeneous if both G_1 and G_2 preserve the structure in question. In this paper we study bihomogeneous manifolds with flat projective structures. This class of manifolds includes S^3, the real projective space $\boldsymbol{RP}^n$ with two hyperplanes removed, and, more generally, the models which can be described as follows.

Let A be an associative algebra with identity over $\boldsymbol{R}$. The open submanifold M of the projective space $P(A)$ corresponding to the open cone of all units in A is bihomogeneous where G_1 and G_2 are the groups of left and right multiplication, respectively. Our first main result is that the universal covers of these models exhaust all simply-connected bihomogeneous projectively flat manifolds.

We reduce the problem to the study of biinvariant affine connections on Lie groups which are projectively flat. We show, in particular, that the only semisimple Lie groups admitting biinvariant flat projective structures are $SL(n, \boldsymbol{R})$ and $SL(n, \boldsymbol{H})$, where $\boldsymbol{H}$ is the field of quaternions. From biinvariant flat projective structures on $SL(n, \boldsymbol{R})$ and $SL(n, \boldsymbol{H})$ we can easily obtain many examples of compact homogeneous

* Partially supported by an NSF grant DMS 8401054. This paper was written while he was a guest research member at the Max-Planck-Institut für Mathematik.