Tôhoku Math. Journ.
39 (1987), 391-406.

A CLASS OF DIFFERENTIAL EQUATIONS OF FUCHSIAN TYPE

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

MAKOTO NAMBA

(Received July 28, 1986)

1. Introduction. The pair of the period integrals

$$Y = \left(\int_\gamma \frac{dz}{w}, \int_\gamma \frac{zdz}{w}\right) \qquad \text{for a 1-cycle } \gamma$$

of the family of elliptic curves

$$w^2 = 4z^3 - xz - y ,$$

parametrized by $(x, y) \in \boldsymbol{C}^2$ with $\Delta = x^3 - 27y^2 \neq 0$, is known to satisfy the following differential equation of Fuchsian type of rank two on the complex projective plane $\boldsymbol{P}^2 = \boldsymbol{P}^2(\boldsymbol{C})$:

$$dY = Y\Omega . \tag{1.1}$$

Here Ω is a (2×2)-matrix-valued meromorphic 1-form on $\boldsymbol{P}^2$ defined by

$$\Omega = \begin{pmatrix} \dfrac{(-1/12)d\Delta}{\Delta} & \dfrac{(3/8)(xydx - (2/3)x^2dy)}{\Delta} \\ \dfrac{(-9/2)(ydx - (2/3)xdy)}{\Delta} & \dfrac{(1/12)d\Delta}{\Delta} \end{pmatrix}.$$

The differential equation (1.1) has regular singularity along $C \cup L_\infty$, where C is the closure in $\boldsymbol{P}^2$ of the affine curve $\{(x, y) \in \boldsymbol{C}^2 \mid \Delta = 0\}$ and L_∞ is the line at infinity.

For $\{\gamma_1, \gamma_2\}$ which gives rise to a $\boldsymbol{Z}$-basis for the first homology group of the elliptic curve with the intersection number $\gamma_1 \cdot \gamma_2 = 1$, the multi-valued map

$$S: \boldsymbol{P}^2 - C \cup L_\infty \to \boldsymbol{C}^2$$

which sends (x, y) to

$$(u, v) = \left(\int_{\gamma_1} \frac{dz}{w}, \int_{\gamma_2} \frac{dz}{w}\right)$$

has the single-valued inverse map

$$S^{-1}: D \to \boldsymbol{P}^2 - C \cup L_\infty ,$$