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# EINSTEIN KAEHLER SUBMANIFOLDS OF A COMPLEX LINEAR OR HYPERBOLIC SPACE

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**Introduction.** Einstein Kaehler submanifolds of a complex space form have been studied by several authors. In the case of codimension one, Smyth [4] and Chern [2] showed them to be either totally geodesic or certain hyperquadrics of a complex projective space. In this classification, Takahashi [5] showed that the Einstein condition can be weakened to the condition that Ricci tensor is parallel. Recently, Tsukada [6] studied the case of codimension two and obtained the same classification. In this paper we completely classify Einstein Kaehler submanifolds of a complex linear or hyperbolic space and prove the following:

THEOREM. *Every Einstein submanifold of a complex linear or hyperbolic space is always totally geodesic.*

Note that our theorem holds for any codimension.

**1. Preliminaries.** It is well-known that the Kaehler metric $g = 2\sum_{\alpha,\beta=1}^{n} g_{\alpha\bar{\beta}} dz^{\alpha} d\bar{z}^{\beta}$ of a Kaehler $n$-manifold $M$ can be locally constructed from a certain real-valued smooth function $f$ by

$$g_{\alpha\bar{\beta}} = \partial^2 f / \partial z^{\alpha} \partial \bar{z}^{\beta} \qquad (\alpha, \beta = 1, \cdots, n)\,,$$

where $(z^1, \cdots, z^n)$ is a local complex coordinate system. Such a function $f$, which is called primitive, is determined up to the real part of a holomorphic function. If the metric $g$ is real analytic, the *diastasis* $D_M(p, q)$ is introduced (cf. [1]), which is a real analytic function defined on a neighborhood of the diagonal set $\{(p, p); p \in M\}$ of the product space $M \times M$ and satisfies the following properties:

(1) The function $D_M(p, q)$ is uniquely determined by the Kaehler metric $g$.

(2) $D_M(p, q) = D_M(q, p)$, and $D_M(p, p) = 0$.

(3) For $p \in M$ fixed, $D_M(p, q)$ is a primitive function of $g$ with respect to the variable $q$.

EXAMPLE 1. Let $(\xi^1, \cdots, \xi^N)$ be the canonical complex coordinate system in $\boldsymbol{C}^N$. Then the diastasis of $\boldsymbol{C}^N$ is given by