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POLARIZED PERIOD MAP FOR GENERALIZED $K3$ SURFACES AND THE MODULI OF EINSTEIN METRICS

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0. Introduction. The moduli space for marked polarized $K3$ surfaces or equivalently the moduli space for marked $K3$ surfaces with a Ricci-flat Einstein-Kähler metric is constructed in [T1] and [L]. This moduli space is isomorphic to an *open dense* subset $K\Omega^0$ of

$$K\Omega := SO_0(3, 19)/SO(2)\times SO(19) .$$

So, it is natural to ask what geometric objects correspond to the "*hole*" $K\Omega\setminus K\Omega^0$ of the moduli space. The purpose of the present paper is to make some contribution to this question from differential geometric point of view. Namely we consider the polarized period map for $K3$ surfaces with simple singular points. The flavor of our main result is most typical in the following:

THEOREM 7. *The moduli space of all Einstein metrics on a $K3$ surface, including Einstein-orbifold-metrics along simple singular points, is isomorphic to*

$$\Gamma\setminus(SO_0(3, 19)/SO(3)\times SO(19)) ,$$

where Γ is the full group of isometries of the $K3$ lattice

$$2(-E_8)\oplus 3\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} .$$

For the proof of this theorem we need two main ingredients, one from algebraic geometry and the other from differential geometry. The algebro-geometric ingredient is the contribution due mainly to Todorov [T1], Looijenga [L], and the generalization of their arguments by Morrison [Mr1] which is very important in the present paper. The differential geometric ingredient is the solution of Calabi's conjecture due to Yau [Ya1] and the equivariant version of it which asserts the existence of a Ricci-flat Einstein-Kähler orbifold-metric on certain complex orbifolds. The existence of a Ricci-flat Einstein-Kähler orbifold-metric makes it possible to use the "*isometric* deformation" of Kähler structures on generalized $K3$ surfaces.