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ANALYTIC MAPPINGS BETWEEN TWO REGULARLY BRANCHED THREE-SHEETED ALGEBROID SURFACES

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

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1. Introduction and results. Baker, Mutō and the author ([1], [4], [5], [6], [8], [9]) have discussed the family of analytic mappings between two ultrahyperelliptic surfaces. In this paper we investigate the structure of the family of analytic mappings between two regularly branched three-sheeted algebroid Riemann surfaces. Here we call a three-sheeted covering Riemann surface regularly branched if it has no branch point other than those of order two.

Let R (resp. S) be the three-sheeted covering algebroid Riemann surface formed by elements $p = (z, y)$ (resp. $q = (w, u)$) for each z, y (resp. w, u) satisfying the equation $y^3 = G(z)$ (resp. $u^3 = g(w)$), where G and g are entire functions, each of which has an infinite number of simple or double zeros and no other zeros. Then, since R and S have branch points of order two only, R and S are regularly branched. If the Nevanlinna counting function $N(r, 0, G)$ for the zeros of G is of finite order $\rho(G)$, then we may assume that G is the canonical product of order $\rho(G)$ over these zeros; a similar remark applies to g.

Let $\mathfrak{A}(R, S)$ denote the family of non-trivial analytic mappings of R into S. Mutō [3] proved:

THEOREM A. *To every $\phi \in \mathfrak{A}(R, S)$ there corresponds a non-constant entire function h such that one of the two functional equations*

$$f_1(z)^3 G(z) = g(h(z))$$

and

$$f_2(z)^3 G(z)^2 = g(h(z))$$

holds, where f_1 is entire and f_2 is a meromorphic function having at most simple poles only at the double zeros of G. The converse is also true.

We call such h the projection for the analytic mapping ϕ and say that

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