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A CANONICAL DECOMPOSITION OF AUTOMORPHIC FORMS WHICH VANISH ON AN INVARIANT MEASURABLE SUBSET

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

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Introduction. Let Γ be a discrete subgroup of the real Möbius group $PSL(2; \boldsymbol{R})$. We denote by $\Omega(\Gamma)$ the region of discontinuity of Γ. Let σ be a Γ-invariant closed subset of the extended real line $\hat{\boldsymbol{R}}$ such that $\sharp\sigma \geqq 3$ and $\sigma \ni \infty$, and let D be the component of $\Omega(\Gamma) - \sigma$ containing the upper half-plane U. Then $D = U$ or $D = \Omega(\Gamma) - \sigma$ according as $\sigma = \hat{\boldsymbol{R}}$ or not. Let E be a Γ-invariant measurable subset of D, and put $V = D - E$, where if $D \neq U$, then E is assumed to be symmetric with respect to $\boldsymbol{R}$ in the sense that $\bar{z} \in E$ whenever $z \in E$. Furthermore, for an integer $q \geqq 2$, let L^p, $1 \leqq p < \infty$, (resp. L^∞) be the Banach space consisting of all the p-integrable (resp. bounded) measurable automorphic forms of weight $-2q$ on D for Γ, which are symmetric if D is symmetric (see Section 1 for the precise definition). We denote by A^p, $1 \leqq p \leqq \infty$, the closed subspace consisting of all the holomorphic elements in L^p, and set $L^p(V) = \{\mu \in L^p; \mu|_E = 0\}$ and $A^p|_V = \{\chi_V\phi; \phi \in A^p\}$, where χ_V is the characteristic function of V. For $1 \leqq p < \infty$ and p' satisfying $1/p + 1/p' = 1$, $L^{p'}$ is isomorphic to the dual space of L^p. We denote by $(A^p)^\perp$ $(\subset L^{p'})$ the annihilator of A^p.

In the present paper, we investigate conditions for E under which $(A^p)^\perp \cap L^{p'}(V)$ and $A^{p'}|_V$ are closed and complementary to each other in $L^{p'}(V)$, and give two kinds of answers to this question (see Theorems 1 and 3 below). This problem occured in studying extremal quasiconformal mappings with dilatation bound (see, for example, Sakan [10]). Our results can be applied to the study of quasiconformal mappings and Teichmüller spaces. These applications will be discussed in Ohtake [9].

Throughout this paper, as natural assumptions for the problem, we require that *V has positive measure and* $A^p \neq \{0\}$. We note that if E has (2-dimensional Lebesgue) measure zero, then the spaces $(A^p)^\perp \cap$

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