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SPECTRAL GEOMETRY OF MINIMAL SURFACES IN THE SPHERE

Dedicated to Professor Luis Esteban Carrasco on his sixty-fifth birthday

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Introduction. Let M be a compact, connected, Riemannian manifold and $f \in C^\infty(M)$ (a smooth function on M). Then we have a spectral decomposition of f, say $f = \sum_{t\geq 0} f_t$, where each f_t is an eigenfunction associated with the eigenvalue λ_t of the Laplacian Δ of M. Certainly f_0 is a constant and the sequence is convergent in L^2-sense.

Now if M is a submanifold in the Euclidean space $\boldsymbol{R}^m$, one has its position vector $x = (x_1, \cdots, x_m)$. So by regarding the spectral decomposition of each x_i one gets the spectral decomposition of x (see (1.1)). If such a spectral decomposition only involves a finite number of nonzero eigenvalues, say k, then the submanifold is said to be of k-type (see Section 1 or [5]). From this point of view the easiest spectral behavior corresponds to the submanifolds of 1-type which are characterized, according to a well known result due to Takahashi [12], as minimal submanifolds in some hypersphere of $\boldsymbol{R}^m$ whose center and radius are completely determined from the center of mass of M into $\boldsymbol{R}^m$ and the associated eigenvalue giving the 1-type character, respectively. Therefore if one wants to study spectral geometry of minimal submanifolds in the sphere, then it seems reasonable to look for the spectral behavior of the products of coordinate functions, $x_i \cdot x_j$, and then to deal with a very special case of the following problem: What is the eigenvalue behavior of the products of eigenfunctions?

In this case one can organize the product of coordinate functions to give a new isometric immersion in the Euclidean space of symmetric matrices over $\boldsymbol{R}$, this is nothing but the composition of the first isometric immersion with the second standard immersion of the sphere in the Euclidean space according to the description given by Sakamoto [11], and then one can study its type number. This idea was used by Ros [10] to give a characterization for minimal submanifolds in the sphere for which the spectral behaviors of $x_i \cdot x_j$ involve exactly two different eigenvalues.

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