Tôhoku Math. Journ.
39 (1987), 557-574.

GLOBAL CONVERGENCE OF SUCCESSIVE APPROXIMATIONS OF SOLUTIONS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY

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(Received September 16, 1986)

1. Introduction. Let E be a Banach space with norm $|\cdot|_E$ and let $J=[\sigma, \sigma+a)$, $0<a \leqq \infty$. If x: $(-\infty, \sigma+a) \rightarrow E$, then for any $t \in (-\infty, \sigma+a)$ we define x_t: $(-\infty, 0] \rightarrow E$ by $x_t(\theta)=x(t+\theta)$, $-\infty<\theta \leqq 0$. In this paper we consider the initial value problem for functional differential equations with infinite delay (IP);

$$\frac{dx}{dt}=f(t, x_t), \quad t>\sigma \tag{1.1}$$

$$x_\sigma=\varphi \in \mathscr{B}, \tag{1.2}$$

where f is an E-valued mapping defined on $J \times \mathscr{B}$ and $\mathscr{B}$ is an abstract phase space with semi-norm or quasi-norm $|\cdot|_{\mathscr{B}}$ satisfying suitable axioms introduced by Hale and Kato [2].

The purpose of this paper is to give sufficient conditions for the global convergence of successive approximations for (IP): main results are Theorems 4.1, 4.2, 4.3 and 4.4, which extend results obtained in [1], [6], [7], [11]. Needless to say, our results ensure the global existence of a unique solution to (IP).

Recently, the author [9] has proved the uniqueness and the global convergence of successive approximations of solutions for functional integral equations under some Perron-type conditions. For (1.1) these conditions become

$$|f(t, \varphi)-f(t, \psi)| \leqq \omega_1(t, |\varphi-\psi|_{\mathscr{B}}) \tag{1.3}$$

and

$$|f(t, x_t)-f(t, y_t)| \leqq \omega_2(t, \sup_{\sigma \leqq s \leqq t} |x(s)-y(s)|), \tag{1.4}$$

where real-valued functions ω_i, $i=1, 2$, are integrable and satisfy some uniqueness conditions. On the other hand, the author [10] has proved Kamke's uniqueness theorems of solutions for (IP). In this paper we shall show the global convergence of successive approximations for (IP)