Tôhoku Math. Journ.
39 (1987), 543-555.

NORMS OF HANKEL OPERATORS AND UNIFORM ALGEBRAS, II

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(Received September 12, 1986)

Abstract. Let H^∞ be an abstract Hardy space associated with a uniform algebra. Denoting by (f) the coset in $(L^\infty)^{-1}/(H^\infty)^{-1}$ of an f in $(L^\infty)^{-1}$, define $\|(f)\|=\inf\{\|g\|_\infty\|g^{-1}\|_\infty;\ g\in(f)\}$ and $\gamma_0=\sup\{\|(f)\|;\ (f)\in(L^\infty)^{-1}/(H^\infty)^{-1}\}$. If γ_0 is finite, we show that the norms of Hankel operators are equivalent to the dual norms of H^1 or the distances of the symbols of Hankel operators from H^∞. If H^∞ is the algebra of bounded analytic functions on a multiply connected domain, then we show that γ_0 is finite and we determine the essential norms of Hankel operators.

0. Introduction. Let X be a compact Hausdorff space, let $C(X)$ be the algebra of complex-valued continuous functions on X, and let A be a uniform algebra on X. For $\tau\in M_A$, the maximal ideal space of A, set $A_0=\{f\in A;\tau(f)=0\}$. Let m be a representing measure for τ on X.

The abstract Hardy space $H^p=H^p(m)$, $1\leqq p\leqq\infty$, determined by A is defined to be the closure of A in $L^p=L^p(m)$ when p is finite and to be the weak*-closure of A in $L^\infty=L^\infty(m)$ when p is infinite. Put $H_0^p=\left\{f\in H^p;\int_X fdm=0\right\}$, $K^p=\left\{f\in L^p;\int_X fgdm=0\text{ for all } g\in A_0\right\}$ and $K_0^p=\left\{f\in K^p;\int_X fdm=0\right\}$. Then $H_0^p\subset K_0^p$ and $H^p\subset K^p$.

Let $Q^{(1)}$ be the orthogonal projection from L^2 to $(H^2)^\perp=\bar{K}_0^2$ and $Q^{(2)}$ the orthogonal projection from L^2 to $\bar{H}_0^2$. For a function ϕ in L^∞ we denote by M_ϕ the multiplication operator on L^2 determined by ϕ. As in the previous paper [14], two generalizations of the classical Hankel operators are defined as follows. For ϕ in L^∞ and f in H^2

$$H_\phi^{(j)}f=Q^{(j)}M_\phi f\quad(j=1,2)\,.$$

If A is a disc algebra and $\tau(f)=\tilde{f}(0)$, where $\tilde{f}$ denotes the holomorphic extension of f in A, then τ is in M_A. The normalized Lebesgue measure m on the unit circle T is a representing measure for τ. Then H^2 is the classical Hardy space and $H_0^2=K_0^2$. Hence $H_\phi^{(1)}=H_\phi^{(2)}$ and it is the classical Hankel operator H_ϕ. It is well known that

* This research was partially supported by Grant-in-Aid for Scientific Research, Ministry of Education.