Tôhoku Math. Journ.
39 (1987), 533-542.

BOUNDED PROJECTIONS ONTO HOLOMORPHIC HARDY SPACES ON PLANAR DOMAINS

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(Received September 12, 1986)

1. Introduction. Throughout this paper, $D \subset \boldsymbol{C}$ is a domain bounded by finitely many non-intersecting simple closed C^4 regular curves. We denote by m_0 the area Hausdorff measure on the boundary ∂D of the domain D, and by m_1 and m_2 two different harmonic measures relative to D. The holomorphic Hardy spaces $H^p(m_j)$ on ∂D are defined as the $L^p(m_j)$-norm closure of $A(\partial D)$ $1 \leqq p < \infty$, where $A(\partial D)$ is the class of continuous functions f on ∂D whose Poisson integral $\mathrm{PI}[f]$ is analytic in D. This paper is concerned with projection operators of $L^p(m_j)$ onto $H^p(m_j)$.

As is well known, there are two bounded projection operators of $L^2(m_j)$ onto $H^2(m_j)$. One of them is the Cauchy projection H and the other is the orthogonal projection P_j. These operators are useful to study real or holomorphic Hardy spaces. In particular, H and P_0 also play important roles in the theory of partial differential equations and of conformal mappings. In addition, P_1 and P_2 are deeply related with uniform algebras.

In this paper, we show correlations between H, P_0, P_1 and P_2, and give some applications to holomorphic Hardy spaces. Our investigation is motivated by the following interesting theorem by Kerzman and Stein [10]:

THEOREM KS ([10]; see also [3]). *Let D be a bounded, simply connected C^∞ domain in the plane, and H^* be the adjoint of H on the Hilbert space $L^2(m_0)$. Then*:

(1) *$H^* - H$ is an integral operator with a smooth kernel. Hence it is compact on $L^2(m_0)$.*

(2) *Further, $I - (H^* - H)$ is an injective bounded operator of $L^2(m_0)$ onto $L^2(m_0)$, and*

$$P_0 = H(I - H^* + H)^{-1} .$$

This result tells us a relation between H and P_0. On the other hand,

* Partly supported by the Grants-in-Aid for Encouragement of Young Scientists, The Ministry of Education, Science and Culture, Japan.