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A RELAXATION THEOREM FOR DIFFERENTIAL INCLUSIONS IN BANACH SPACES

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Abstract. In this paper we consider differential inclusions in a separable Banach space and we show that when the orientor field satisfies the Caratheodory conditions and is Lipschitzean with respect to a Kamke function $w(t, x)$ in the state variable, then the set of solutions of the nonconvex problem is dense in the $C_X(T)$-topology in the set of solutions of the convexified problem.

Introduction. For a differential inclusion $\dot{x}(t) \in F(t, x(t))$ with a nonconvex right hand side the set of solutions through an initial point is, in general, not closed (even if its sections might be). So we would like to know what is the relation of the closure of this set to the set of the solutions of the convexified problem. This problem was first considered by Ważewski [20] who introduced the notion of a quasitrajectory and proved that whenever $F(t, x)$ is continuous, every solution $\dot{x}(t) \in \mathrm{clconv}\, F(t, x(t))$ is the limit of a sequence of quasitrajectories of $\dot{x}(t) \in F(t, x(t))$. However such a result does not provide an estimate of the distance between a quasitrajectory and a true solution. In fact such an estimate cannot be obtained if we only assume that $F(\cdot, \cdot)$ is continuous. The additional condition needed is a Lipschitzness condition in the x-variable of $F(\cdot, \cdot)$. With that condition present, Filippov [5] was able to obtain the missing estimate and then prove the desired density result. A very nice presentation of those results can be found in the book of Clarke [3, pp. 115-118]. Later Pliss [12] provided a counterexample which illustrated that the Lipschitzness condition cannot be omitted. A generalization of Filippov's theorem was given by Pianigiani [11]. However all these results were for $\boldsymbol{R}^n$. The only infinite dimensional relaxation result that we know of, is that of Tolstonogov [17, Theorem 4.3], which was stated though without a proof. Here we present another such theorem, with a different set of hypotheses. The motivation for such an infinite dimensional result comes from the optimal control theory of systems governed by an evolution equation (distributed parameter systems, see for example Ahmed-Teo [1]).

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