

SECOND ORDER DIFFERENTIAL OPERATORS AND DIRICHLET INTEGRALS WITH SINGULAR COEFFICIENTS:

I. FUNCTIONAL CALCULUS OF ONE-DIMENSIONAL OPERATORS

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Contents

Introduction	466
Chapter I. Definition of operators with singular coefficients and their applications.....	466
1. Motivations coming from mathematical physics problems	467
2. Relation with the general theory of Dirichlet integrals	469
3. Definition of the operator L	469
4. The one dimensional case: method of solution.....	469
Chapter II. The case of piecewise constant coefficients.....	472
1. Hypothesis and general formulas for the transfer matrix	472
2. The self-adjoint case	473
3. The non self-adjoint case	475
4. The particular cases $N=2$ or 3 : self-adjoint cases.....	475
5. The particular cases $N=2$ or 3 : non self-adjoint cases	478
Chapter III. The operator with general irregular coefficients.....	480
1. Computing a finite product of transfer matrices	480
2. The heat kernel for a general finite N	483
3. Going to the continuum limit: case of continuous coefficients	484
4. The continuum limit: case of discontinuous coefficients	488
5. Comments about the form of the Green functions.....	491
Chapter IV. An example of singular perturbation: limit of operators with irregular coefficients.....	492
1. An example of a sequence of operators and their heat kernels	492
2. The case: μ tends to 1	493
3. The case: μ tends to μ_0 , $-1 < \mu_0 < +1$	494
4. The case: μ tends to -1	495
5. Conclusion	495
Chapter V. Diffusion operators with spherical symmetry in \mathbb{R}^3	496
1. Transfer matrix for a self-adjoint operator with piecewise coefficients	496
2. Spectral resolution for a self-adjoint operator with piecewise constant coefficients.....	500
3. Spectral resolution for a general self-adjoint operator (continuous coefficients).....	501
References	504