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GEVREY HYPOELLIPTICITY OF A CLASS OF PSEUDODIFFERENTIAL OPERATORS

Dedicated to Professor Tadashi Kuroda on his sixtieth birthday

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Introduction. We consider a pseudodifferential equation:

$$(1) \qquad a(x, D)u = f$$

with data f having Gevrey index $s \geqq 1$. Here $a(x, D)$ is a pseudodifferential operator of type $S^m_{\rho,\delta}$ of Hörmander (cf. [4]). We are interested in the Gevrey regularity of solutions, more precisely, in which way the Gevrey index of solutions depends on ρ, δ and s.

In [3], we have given the definition of a class of hypoelliptic pseudodifferential operators of symbol class $S^m_{\rho,\delta,\sigma}(\Omega\times R^n)$, $\Omega\subset R^n$, $0\leqq\delta<\rho\leqq 1$, $\sigma\geqq 1$, which consists of symbols $a(x, \xi)\in S^m_{\rho,\delta}(\Omega\times R^n)$ satisfying

$$(2) \qquad |a(x, \xi)| \geqq c|\xi|^{m'}, \quad |\xi|\geqq B, \ -\infty < m' < \infty ,$$

$$(3) \qquad |a^{(\alpha)}_{(\beta)}(x, \xi)| \leqq C_0 C_1^{|\alpha+\beta|}\alpha!\,\beta!^{\sigma}|a(x, \xi)|(1+|\xi|)^{-\rho|\alpha|+\delta|\beta|} , \quad x\in\Omega,\ |\xi|\geqq B|\alpha|^{\theta},\ \theta=\sigma/(\rho-\delta) .$$

Under these conditions, we have constructed a parametrix b of $a(x, D)$ with symbol $b(x, \xi)\in S^{-m'}_{\rho,\delta,\sigma}(\Omega\times R^n)$. Here b is expressed by an infinite series of symbols, and the remainder $r = ba - I$ is an integral operator with a kernel of Gevrey function of index $\theta=\sigma/(\rho-\delta)$ (cf. Theorem 3.1 and Corollary 3.1 of [3]). Thus we have $\max(\sigma/(\rho-\delta), s)$ as the Gevrey index for solutions of the equation (1). This gives the best possible index when $\rho\equiv 1$, $0\leqq\delta<1$ as was shown by several examples in [3], but not necessarily the best possible when $0<\rho<1$.

It seems impossible to apply directly the method of [3] to obtain sharper results if $0<\rho<1$. We use a finite approximation of parametrix instead of infinite approximation used in [3]. The remainder term is not necessarily smooth, so we are forced to estimate all derivatives of solutions inductively. This method seems unusual in the study of hypoellipticity because it looks tedious. However, surprisingly this method provides a sharper result for Gevrey hypoellipticity. For the nonlinear problem such a method was used by Friedman [2] to get