Tôhoku Math. J.
40 (1988), 473-483.

POSITIVE KERNEL FUNCTIONS AND BERGMAN SPACES

NOZOMU MOCHIZUKI

(Received May 8, 1987)

Introduction. We denote by B theo pen unit ball in C^n, $n \geqq 1$. The Poisson kernel for B is obtained from the Cauchy kernel. In the same way, we can define a positive kernel function, H_δ, from the well-known kernel which is treated in [5]. H_δ has the reproducing property for the functions in the weighted Bergman space $A^{p,\delta}(B)$, $1 \leqq p < +\infty$. Using this kernel we shall derive Hardy-Littlewood inequalities for $A^{p,\delta}(B)$, just as in $H^p(B)$, where the Poisson kernel plays an essential role ([7]). Similar results will be obtained in the setting of the generalized half plane in C^n. As an application of the inequality, we shall treat the Mackey topology of $A^{p,\delta}(B)$, $0 < p < 1$, extending the one variable result ([9]).

1. Positive kernels. $\langle z, w \rangle$ will denote the usual inner product for $z, w \in C^n$ with $|z|^2 = \langle z, z \rangle$. We fix $\delta > -1$ throughout. Let $K_\delta(z, w) = A_0(1 - |w|^2)^\delta(1 - \langle z, w \rangle)^{-(n+1+\delta)}$, $z, w \in B$, where

$$A_0 = \left(\int_B (1 - |w|^2)^\delta dw\right)^{-1} = \frac{\Gamma(n + 1 + \delta)}{\Gamma(1 + \delta)\pi^n} ;$$

here, dw denotes Lebesgue measure on R^{2n}. We define a positive kernel H_δ by

$$H_\delta(z, w) := \frac{K_\delta(z, w)K_\delta(w, z)}{K_\delta(z, z)} = \frac{A_0(1 - |z|^2)^{n+1+\delta}(1 - |w|^2)^\delta}{|1 - \langle z, w \rangle|^{2(n+1+\delta)}} , \quad z, w \in B .$$

We shall write

$$H_\delta[f](z) = \int_B H_\delta(z, w)f(w)dw , \quad z \in B ,$$

when the integral makes sense. For $0 < p < +\infty$, $L^{p,\delta}(B)$ will denote the class of measurable functions f on B such that

$$\|f\|_{p,\delta} := \left(\int_B |f(w)|^p(1 - |w|^2)^\delta dw\right)^{1/p} < +\infty ,$$

and $A^{p,\delta}(B)$ will mean the class of holomorphic functions which belong to

---

Partially supported by the Grant-in-Aid for Scientific Research, the Ministry of Education, Science and Culture, Japan.