**VECTOR BUNDLES OVER QUATERNIONIC KÄHLER MANIFOLDS**

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**Introduction.** On vector bundles over oriented 4-dimensional Riemannian manifolds, the notion of self-dual and anti-self-dual connections plays an important role in the geometry of 4-dimensional Yang-Mills theory (see Atiyah, Hitchin and Singer [A-H-S]).

On the other hand, in his differential-geometric study of stable holomorphic vector bundles, Kobayashi [K] introduced the concept of Einstein-Hermitian vector bundles over Kähler manifolds. Let $E$ be a vector bundle over a quaternionic Kähler manifold $M$, and $p: Z \to M$ the corresponding twistor space defined by Salamon [S1]. Now the purpose of the present paper is to give a quaternionic Kähler analogue of self-dual and anti-self-dual connections, and then to construct a natural correspondence between $E$'s with such connections and the set of Einstein-Hermitian vector bundles over $Z$.

Let $H$ be the skew field of quaternions. Then the $Sp(n) \cdot Sp(1)$-module $\Lambda^2 H^*$ is a direct sum $N'_2 \oplus N''_2 \oplus L_2$ of its irreducible submodules $N'_2$, $N''_2$, $L_2$, where $N'_2$ (resp. $L_2$) is the submodule of the elements fixed by $Sp(n)$ (resp. $Sp(1)$) and for $n = 1$, we have $N''_1 = \{0\}$. Hence, the vector bundle $\Lambda^2 T^* M$ is written as a direct sum $A'_2 \oplus A''_2 \oplus B_2$ of its holonomy-invariant subbundles in such a way that $A'_2$, $A''_2$, $B_2$ correspond respectively to $N'_2$, $N''_2$, $L_2$. Now, a connection for $E$ is called an $A'_2$-connection (resp. $B_2$-connection) if the corresponding curvature is an $\text{End}(E)$-valued $A'_2$-form (resp. $B_2$-form). Then we have:

**Theorem (0.1).** All $A'_2$-connections and also all $B_2$-connections are Yang-Mills connections.

Furthermore, for $E$ with a $B_2$-connection we can associate an $E$-valued elliptic complex (cf. (3.2)) similar to those of Salamon [S2]. Such complexes allow us to analyze the space of infinitesimal deformations of $B_2$-connections (see Theorem (3.5)).

For our quaternionic Kähler manifold $M$, a pair $(E, D_E)$ of a vector bundle $E$ over $M$ and a $B_2$-connection $D_E$ on $E$ is called a Hermitian pair on $M$ if $D_E$ is a Hermitian connection on $E$. On the other hand, a pair $(F, D_F)$ of a holomorphic vector bundle over $Z$ and a Hermitian $(1, 0)$-