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ON THE GENERALIZATION OF FROSTMAN'S THEOREM DUE TO S. KOBAYASHI

To Professor Tadashi Kuroda on the occasion of his sixtieth birthday

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1. For a single-valued meromorphic function $f(z)$ in a domain D of the z-plane and a boundary point ζ of D, the range of values $R_D(f, \zeta)$ of f at ζ is defined by $R_D(f, \zeta) = \cap_{r>0} f(D \cap U(\zeta, r))$, where $U(\zeta, r)$ denotes the open disc $|z-\zeta| < r$. We denote by $H_{|f|}(z)$ and $H_{|f|^2}(z)$ the least harmonic majorants of $|f(z)|$ and $|f(z)|^2$ in D, respectively.

In the case where D is the unit disc, it is known as Frostman's theorem [1] that if $|f(z)| < 1$ in $|z| < 1$ and Fatou's boundary function f^* of f satisfies $|f^*(\eta)| = 1$ almost everywhere on $|\eta| = 1$ and if f is not analytic at ζ, $|\zeta| = 1$, then $R_{|z|<1}(f, \zeta)$ covers the unit disc $|w| < 1$ except possibly for a set of capacity zero, where capacity means logarithmic capacity. In this case $H_{|f|}(z) = H_{|f|^2}(z) \equiv 1$ in $|z| < 1$ and the assumption that f is not analytic at ζ is equivalent to the existence of a sequence $\{z_n\}$ of points in $|z| < 1$ converging to ζ with $\lim_{n\to\infty} f(z_n) = 0$.

Recently, as a generalization of the above theorem to the case of general domains, Kobayashi [2] has given the following theorem.

THEOREM. *Suppose that $|f(z)| < 1$ in D and that $\zeta \in \partial D$ is a regular boundary point with respect to the Dirichlet problem. If there exists a sequence $\{z_n\}$ of points in D converging to ζ for which $H_{|f|^2}(z_n) \to 1$ and $f(z_n) \to a$ with $|a| < 1$ as $n \to \infty$, then $R_D(f, \zeta)$ covers the unit disc except possibly for a set of capacity zero.*

Our aim of the present note is to show that the standard argument in the theory of cluster sets gives a much simpler proof of Kobayashi's theorem and includes the case where ζ is an irregular boundary point. We shall prove:

THEOREM. *Suppose that $|f(z)| < 1$ in D and that there exists a sequence $\{z_n\}$ of points in D converging to $\zeta \in \partial D$ for which $H_{|f|}(z_n) \to 1$ and*

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