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TRANSLATION AND CANCELLATION OF SOCLE SERIES PATTERNS*

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Let G be a simply connected semisimple linear algebraic group over an algebraically closed field K of characteristic $p>0$, B a Borel subgroup of G, and T a maximal torus of B. Let G_n be the n-th Frobenius kernel of G, and G_nB the closed subgroup scheme of G generated by G_n and B. Then we have induction functors $\hat{Z}_n = \mathrm{Ind}_B^{G_nB}$, $\mathrm{Ind}_{G_nB}^G$ and Ind_B^G. The first one of these functors is exact, but the others are only left exact, so we can further construct their right derived functors $H_n^i = R^i\,\mathrm{Ind}_{G_nB}^G$ and $H^i = R^i\,\mathrm{Ind}_B^G$ for all $i \geqq 0$. Thanks to the transitivity of inductions we obtain that $H^0 = H_n^0 \circ \hat{Z}_n$, or more generally, $H^i = H_n^i \circ \hat{Z}_n$.

Let $X(T)$ be the character group of T. Let $\lambda \in X(T)$, canonically regarded as a 1-dimensional B-module. In this paper we shall reveal a connection between the G_n-socle series of $\hat{Z}_n(\lambda)$ and the G-socle series of $H^0(\lambda)$. Our main result (cf. (2.1)) is a generalization of a similar result of Andersen (cf. [3, (4.4)]). As an application of the main result, we shall also discuss the G-socle series of $H^0(\lambda)$ for non-generic λ in the B_2 case.

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1. Preliminaries. Let R be the root system of G with respect T, and W its Weyl group. Choose the set of positive roots, denoted by R_+, such that B corresponds to $-R_+$, and denote the set of simple roots by S. Let E be the Euclidean space, spanned by R with W-invariant inner product $\langle\ ,\ \rangle$. Then we can identify $X(T)$ with the abstract weight lattice of R. Let $X(T)_+$ be the set of dominant weights with respect to the

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