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COMPLETE NEGATIVELY PINCHED KÄHLER SURFACES OF FINITE VOLUME

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0. Introduction. Recently many differential geometers are interested in complete Riemannian manifolds of negative or nonpositive curvature. On the other hand, complete Kähler manifolds of negative curvature are interesting objects in complex analysis because of their function-theoretic properties. But few results are known about them. Moreover, in general it is hard to construct examples of negatively curved Kähler manifolds of finite volume. A typical example is an arithmetic quotient of a bounded symmetric domain of rank one.

In this paper, we shall investigate the 2-dimensional case. The purpose of this paper is to study how the differential geometric properties reflect the complex structures in the case of complete negatively pinched Kähler surfaces of finite volume. More precisely, we study complete Kähler surfaces S such that

1. $\mathrm{vol}(S) < \infty$,
2. $-1 \leqq c^-(S) \leqq c^+(S) < 0$,

where $c^+(S)$, $c^-(S)$ denote the supremum and the infimum of the sectional curvatures of S, respectively. By [14, p. 363, Main Theorem], such a complete Kähler surface S is a quasi-projective surface and can be compactified as a normal projective surface $\hat{S}$ by addition of one point to each end. We call $\hat{S}$ the Siu-Yau compactification of S. In this paper, we shall show that $\hat{S}$ has only almost simple elliptic singularities. In other words, we can determine the complex structure at infinity of S. Our proof depends on the classification of normal surface singularities with solvable local fundamental groups ([16]).

This paper is organized as follows. In Section 1, we determine the minimal resolution of the normal isolated singularities of $\hat{S}$ by using Wagreich's classification of normal isolated surface singularities with solvable local fundamental groups ([16]). In Section 2, we study the asymptotic behavior of the Kähler metric of S toward infinity by using the existence of a complete Kähler-Einstein metric with negative scalar curvature ([6]). In Section 3, we prove a finiteness theorem of deformation types if we pinch the curvature and volume of the surfaces. In Section 4, we remark