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REPRESENTATION OF THE SOLUTION OPERATOR GENERATED BY FUNCTIONAL DIFFERENTIAL EQUATIONS

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1. Introduction. Hale gave a representation theorem for the solution operator generated by a neutral functional differential equation (NFDE) with finite delay, which represents the solution operator as the sum of a bounded linear operator with zero spectrum and a conditionally completely continuous operator (cf. [1]). This result proved to be very useful in studying the existence of periodic solutions of NFDE, and has been generalized to functional differential equations (FDE) with infinite delay (cf. [2]). In this paper, we generalize the latter result to NFDE with infinite delay, and give some applications to the existence of periodic solutions.

In the present paper, we denote the segment of a function $x(s)$ for $-\infty<s\leqslant t$ by x_t, and let X be a Banach space of some real functions $\phi:(-\infty, 0]\to R^n$ with the norm $\|\phi\|$ having the following properties:

(H_1) If $x:(-\infty, \sigma+A)\to R^n$, $A>0$, $\sigma\geqslant 0$, is continuous for $t\in[\sigma, \sigma+A)$ and $x_\sigma\in X$, then $x_t\in X$ and x_t is continuous for $t\in[\sigma, \sigma+A)$.

(H_2) There is a positive constant k_0 such that $|\phi(0)|\leqslant k_0\|\phi\|$, for $\phi\in X$, where $|\cdot|$ stands for a norm in R^n.

(H_3) There are positive constants K and M such that if x satisfies (H_1) then

$$\|x_t\|\leqslant K\sup_{u\in[\sigma,t]}|x(u)|+M\|x_\sigma\|\,,\qquad t\geqslant\sigma\,.$$

A continuous functional $D:[0,\infty)\times X\to R^n$ is said to be atomic (cf. [3]), if it can be represented as

$$D(t)\phi=A(t)\phi(0)-L(t)\phi\,,\qquad t\geqslant 0\,,\quad \phi\in X\,,$$

with a continuous nonsingular $n\times n$ matrix $A(t)$ and a bounded linear operator $L(t): X\to R^n$ which satisfy $\sup_{t\geqslant 0}|L(t)|\leqslant L$, $\sup_{t\geqslant 0}(|A(t)|+|A^{-1}(t)|)\leq A$ and $|L(t)\phi|\leqslant\gamma(\beta)\|\phi\|$ for $t\geqslant 0$, $\beta\geqslant 0$ and $\phi\in X$ with compact support contained in $(-\infty, 0]$, where L and A are positive constants and γ is a nonnegative continuous function on $[0,\infty)$ with $\gamma(0)=0$. Here and hereafter, $|L(t)|$ and $|A(t)|$ stand for the operator norms of $L(t)$ and $A(t)$, respectively.

For any atomic D and any $H\in C([\sigma,\infty), R^n)$, the equation

$$D(t)z_t=H(t)\,,\quad t\geqslant\sigma\geqslant 0\,,\quad z_\sigma=\phi\in X\,,\quad H(\sigma)=D(\sigma)\phi\,, \tag{1.1}$$