Tôhoku Math. J.
42 (1990), 67–82

THE GAUSS MAP AND SPACELIKE SURFACES WITH PRESCRIBED MEAN CURVATURE IN MINKOWSKI 3-SPACE

KAZUO AKUTAGAWA AND SEIKI NISHIKAWA*

(Received November 18, 1988)

For an oriented spacelike surface M in Minkowski 3-space L^3, the Gauss map G is defined to be a mapping of M into the unit pseudosphere $\boldsymbol{H}$ in L^3, which assigns to each point p of M the point in $\boldsymbol{H}$ obtained by translating the timelike unit normal vector at p to the origin. Our primary object of this paper is to prove a representation formula for spacelike surfaces with prescribed mean curvature in terms of their Gauss maps.

It is well-known that the classical Weierstrass-Enneper representation formula describes minimal surfaces in Euclidean 3-space $\boldsymbol{R}^3$ in terms of their Gauss maps and auxiliary holomorphic functions ([8]). More generally, a remarkable representation formula has been discovered by Kenmotsu [3] for arbitrary surfaces in $\boldsymbol{R}^3$ with nonvanishing mean curvature, which describes these surfaces in terms of their Gauss maps and mean curvature functions. On the other hand, Kobayashi [4, 5] proved the Lorentzian version of the classical Weierstrass-Enneper representation formula for maximal surfaces in Minkowski 3-space L^3 (see also McNertney [10]) and applied it to the study of maximal surfaces with conelike singularities.

Motivated by these results, we shall prove, in §4 of this paper, that arbitrary oriented spacelike surfaces in L^3 satisfy a system of first order partial differential equations involving the mean curvature function H and the Gauss map G of the surface (Theorem 4.1). An interesting feature therein is that the complete integrability condition for the formula then yields a system of nonlinear second order partial differential equations which identifies the gradient of H and the tension field of G (Proposition 5.3). In particular, the condition simply means that the Gauss map G should be a harmonic mapping provided the mean curvature H is constant.

The converse of these observations will be discussed in §6. Our main result is that given a nowhere holomorphic smooth mapping G of a simply connected Riemman surface M into the pseudosphere $\boldsymbol{H}$ satisfying the complete integrability condition for some nonvanishing smooth function H on M, we can construct explicitly a spacelike immersion of M into L^3 such that the mean curvature of M is H and the Gauss map of M is given by G (Theorem 6.1). This allows us, in particular, to produce a wealth of spacelike surfaces of constant mean curvature in L^3, and more importantly, to relate

* Partially supported by the Grant-in-Aid for Scientific Research No. 60540058.