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VARIETIES WHOSE SURFACE SECTIONS ARE ELLIPTIC

HARRY D'SOUZA* AND MARIA LUCIA FANIA**

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Introduction. Let X be a complex projective manifold of dimension n. Let L be a *very ample* line bundle on X. Let S be the intersection of $n-2$ general members of $|L|$. Assume, moreover, that S is an *elliptic surface of Kodaira dimension* $\kappa(S)=1$ and that (X, L) is not a scroll over a surface. By [20], we know that there exists a reduction (X', L') of (X, L) such that $K_{X'}+(n-1)L'$ is very ample. Moreover, $K_{X'}+(n-2)L'$ is semi-ample and any smooth surface S', which is the intersection of $n-2$ general members of $|L'|$ is a *minimal model*, with $\kappa(S')\geq 0$ (see [9], [13], [19]). Let $p\colon X'\to C$ be the morphism associated to $|N(K_{X'}+(n-2)L')|$ for $N\gg 0$. N is chosen so that $C=p(X')$ is normal and p has connected fibres. It follows that $\dim C=1$. We restrict ourselves to varieties of dimension $n\geq 4$, since the case $n=3$ has been considered by the first author in [2]. Note that the general fibre of p is a del Pezzo manifold of degree d, where $3\leq d\leq 8$. We classify X' in the cases $d=3, 4, 7, 8$. Since we have only partial results for $d=5, 6$ those will not be included here.

The paper is organized as follows. In Section 0, we give some background material and state, without proof, some of the needed results. In Section 1, we prove the results used later in the paper. In Section 2 we classify the possible singular fibres of the morphism $p_3\colon X'^3\to C$ in the case $d=3, 4, 7, 8$. In Sections 3 through 6 we analyze the structure of X' for these values of d.

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0. Notation and background material. Throughout this paper we let X be an irreducible complex projective manifold of dimension n, and L a very ample line bundle over X.

(0.1) Let L be a line bundle over X. We say that L *is nef* if $c_1(L)\cdot[C]\geq 0$, for all effective curves C on X. We say that a nef line bundle L is *big* if $c_1(L)^n>0$. We say

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