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SOME p-GROUPS WITH TWO GENERATORS WHICH SATISFY CERTAIN CONDITIONS ARISING FROM ARITHMETIC IN IMAGINARY QUADRATIC FIELDS

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Abstract. Let p be an odd prime. We give a list of certain types of p-groups G with two generators which satisfy the following two conditions (A) and (B): (A) $[\mathrm{Ker}\, V_{G\to H}:[G, G]]=[G:H]$ for the transfer homomorphism $V_{G\to H}: G\to H/[H, H]$ of G to every normal subgroup H with cyclic quotient G/H, and (B) there exists an automorphism φ of G of order 2 such that $g^{\varphi+1}\in[G, G]$ for every $g\in G$. These conditions are necessary for G to be the Galois group of the second p-class field of an imaginary quadratic field. The list contains such a group that it may be useful for us to find an imaginary quadratic field with an interesting property on the capitulation problem.

1. Introduction. We fix an odd prime number p, and consider a finite metabelian p-group G with two generators which satisfies the following two conditions (A) and (B):

(A) For every normal subgroup H of G with cyclic quotient G/H, the index $[\mathrm{Ker}\, V_{G\to H}:[G, G]]$ for the transfer homomorphism $V_{G\to H}: G\to H/[H, H]$ coincides with the index $[G:H]$;

(B) There exists an automorphism φ of G of order 2 such that $g^{\varphi+1}$ belongs to $[G, G]$ for every $g\in G$.

It is known as Hilbert's Theorem 94 that the former index in (A) is a multiple of the latter (cf. Suzuki [Su] for the general case); therefore, the condition (A) claims the extreme for the normal subgroups H of the kind. If G is abelian, then it satisfies (A) if and only if it is a cyclic group. It is also easy to see that G is not a metacyclic group if it satisfies the condition (B). On his list in [Ja], roughly and boldly speaking, James gave bout 500 types of p-groups of order up to p^6; however, we find only 8 of them with two generators satisfy both of (A) and (B).

In this paper we determine all of such groups of the following form with either one of the three conditions, (1) $m=n=2$, (2) $\mu=1\leq\nu$, and (3) $\mu=\nu=2$ (Theorems 1–3 in Section 7, respectively):

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