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DISTRIBUTION OF RATIONAL POINTS ON HYPERELLIPTIC SURFACES

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Abstract. In this paper, we study distribution of rational points on a hyperelliptic surface defined over an algebraic number field, and show that this distribution is very similar to the distribution of rational points on an abelian surface. As an application, we show that a conjecture of Batyrev-Manin holds for such a surface.

1. Introduction. Let k be an algebraic number field of finite degree and V a nonsingular projective variety defined over k. It is one of the most important problems in number theory to study the set $V(k)$ of k-rational points on V.

One can study the structure of $V(k)$, especially the distribution of k-rational points on V, by using height functions in the following way:

Let $\mathscr{L}$ be an ample invertible sheaf on the k-variety V and let $h_{\mathscr{L}}$ be the (absolute) logarithmic height function associated to $\mathscr{L}$. Then, for any positive number M, $\{P\in V(k); h_{\mathscr{L}}(P)\leq M\}$ is a finite set. We define a function $N_{\mathscr{L}}(V(k); M)$ of M by

$$N_{\mathscr{L}}(V(k); M)=\sharp\{P\in V(k);\ h_{\mathscr{L}}(P)\leq M\}\ .$$

On can obtain very important information on $V(k)$ by investigating the asymptotic behavior of $N_{\mathscr{L}}(V(k); M)$ as $M\to\infty$.

Let A be an abelian variety defined over k. Then the set $A(k)$ of k-rational points on A is a finitely generated abelian group (the Mordell-Weil theorem). In 1965, Néron [7] obtained the following asymptotic formula by using the canonical height:

$$N_{\mathscr{L}}(A(k); M)=cM^{r/2}+O(M^{(r-1)/2})\qquad \text{as}\quad M\to\infty\ ,$$

where r is the rank of the abelian group $A(k)$ and c is a positive number which depends only on the algebraic equivalence class of $\mathscr{L}$.

For the n-dimensional projective space $\boldsymbol{P}^n$, Schanuel [8] obtained the following asymptotic formula:

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