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THE MORDELL-WEIL GROUP OF CERTAIN ABELIAN VARIETIES DEFINED OVER THE RATIONAL FUNCTION FIELD

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Abstract. An explicit method of construction of a family of abelian varieties each member of which has a large Mordell-Weil rank is given. Also, an example of elliptic curve defined over a function field of one variable such that its Mordell-Weil group is of arbitrarily high rank is constructed.

Introduction. In our earlier paper [3], we proved the following theorem:

THEOREM 0.1. *Let C be a hyperelliptic curve over a field k and let A be an abelian variety over k. Let A_b denote the twist of A by the quadratic extension $k(C)/k(\boldsymbol{P}^1)$ so that A_b is an abelian variety over $k(\boldsymbol{P}^1)=k(t)$. Then we have an isomorphism of abelian groups*

$$A_b(k(t))\cong \mathrm{Hom}_k(J(C), A)\oplus A_2(k)\,,$$

where $A_2(k)$ denotes the group of k-rational 2-division points on A.

In PART A of this paper we investigate what occurs if one specializes the value of t in (0.1) (see Theorem 2.1). This enables one to reduce the problem of the injectivity of the specialization map of the family to that of the unsolvability of a certain Diophantine equation. Such examples are given in Section 3. In particular, we obtain an example of a family E_t of elliptic curves over $\boldsymbol{P}^1$ such that for any $t\in \boldsymbol{P}^1(\boldsymbol{Q})-\{0, \pm 1, \infty\}$, the Mordell-Weil group $E_t(\boldsymbol{Q})$ has rank ≥ 2. In PART B we formulate a generalization of Theorem 0.1 to the case of arbitrary double coverings (see Theorem 4.1). As a corollary, we obtain an elliptic curve E defined over the function field of a curve C over $\boldsymbol{Q}$ such that its Mordell-Weil group $E(\boldsymbol{Q}(C))$ is of arbitrarily high rank (see Theorem 4.5). For the construction, we use certain modular curves and its *Atkin-Lehner involutions.*

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