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GLOBAL UNIQUENESS FOR OVALOIDS IN EUCLIDEAN AND AFFINE DIFFERENTIAL GEOMETRY

UDO SIMON*

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Abstract. Ovaloids are uniquely determined by the connections induced from a relative normalization, and the volume form of the relative metric. While the equiaffine interpretations are new, the Euclidean specialization revisits results of Minkowski, Liebmann and Cohn-Vossen.

1. Introduction. From Bonnet's theorem two surfaces $x, x^{\#}: M \to E_3$ in Euclidean 3-space are equivalent modulo a Euclidean motion if the first and second fundamental forms I, II coincide on M:

$$\mathrm{I} = \mathrm{I}^{\#}, \quad \mathrm{II} = \mathrm{II}^{\#}.$$

If the Euclidean Gauss curvature K is non zero, one can state local analogues using two of the three fundamental forms I, II, III of the surfaces.

There are a series of well-known global uniqueness results for ovaloids. In (1.1)–(1.3) we recall three of them. We state the assumptions which imply the equivalence of $x, x^{\#}$ modulo Euclidean motions:

(1.1) MINKOWSKI 1903: $\mathrm{III} = \mathrm{III}^{\#}$, $K = K^{\#}$.

(1.2) COHN-VOSSEN 1927: $\mathrm{I} = \mathrm{I}^{\#}$. LIEBMANN proved in 1901 a corresponding result about infinitesimal rigidity.

(1.3) GROVE: $\mathrm{II} = \mathrm{II}^{\#}$, $K = K^{\#}$.

In [H et al] we collected different methods of proof for these results and generalizations due to various authors; references are included there.

The equiaffine analogue to Bonnet's local theorem is Radon's existence and uniqueness result, which similarly holds in relative differential geometry ([BLA, §60, 65]; [SCHI, Chap. IV, V, VIII]). Barthel proved a version of the fundamental theorem emphazising the role of the induced first connection; this and some global uniqueness results for relative connections are considered in [SI-1].

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