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ADMISSIBLE SOLUTIONS OF SECOND ORDER DIFFERENTIAL EQUATIONS

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Abstract. We treat second order differential equations which have admissible meromorphic solutions. With the aid of Nevanlinna theory, we obtain generalizations of the celebrated theorem of Malmquist-Yosida.

1. Introduction. We will treat differential equations of second order

$$w''=F(z, w, w'), \tag{1.1}$$

where F is a polynomial in w and w' with meromorphic coefficients.

There are famous theorems due to Painlevé, Malmquist, Yosida and others for the analytic theory of ordinary differential equations.

Painlevé classified the equation (1.1) according to the nature of their singularities. Fixed singularities can arise at the locations of singularities of the coefficients. Singularities that are not fixed are said to be movable. Painlevé and his collaborators found six equations whose solutions do not have movable singularities except poles. They are known as the Painlevé transcendents and have a great variety of interesting properties (see [13, pp. 294–298] or [24, pp. 375–377]).

On the other hand, Malmquist investigated equations which possess meromorphic solutions. With the aid of Nevanlinna theory, Yosida [26] generalized the theorem of Malmquist, which is the starting point in this field.

THEOREM A (Malmquist-Yosida). *Let $R(z, w)$ be a rational function in z and w. If the differential equation*

$$(w')^p=R(z, w) \tag{1.2}$$

possesses a transcendental meromorphic solution, then $R(z, w)$ must be a polynomial in w of degree at most $2p$.

Then, several mathematicians treated the differential equations with the aid of Nevanlinna theory, and many generalizations of this theorem have been obtained, for example [7], [16]. In particular, equations of second order have been investigated in [18], [21]–[23], [25]. Steinmetz [21] treated the equation

$$w''=Q(z, w)w'+P(z, w), \tag{1.3}$$

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