Tôhoku Math. J.
44 (1992), 523–534

PARAMETER SHIFT IN NORMAL GENERALIZED HYPERGEOMETRIC SYSTEMS

MUTSUMI SAITO*

(Received October 7, 1991, revised April 28, 1992)

Abstract. We treat the problem of shifting parameters of the generalized hypergeometric systems defined by Gelfand when their associated toric varieties are normal. In this context we define and determine the Bernstein-Sato polynomials for the natural morphisms of shifting parameters. We also give some examples.

Let $A=\{\chi_1, \dots, \chi_N\} \subset \boldsymbol{Z}^n$ be a finite subset with certain properties. In [G], [GGZ], [GZK1], [GZK2], [GKZ] and so on, Gelfand and his collaborators defined and studied generalized hypergeometric systems M_α associated to A with parameter α. Aomoto defined and studied a broader class of systems (cf. [A1]–[A4]). Generalized hypergeometric systems of this kind were also defined in [KKM] and [H], where they were named canonical systems. For $1 \leq j \leq N$, there exists a natural morphism $f_{\chi_j}: M_{\alpha-\chi_j} \to M_\alpha$, which corresponds to the differentiation of solutions. In this paper, we treat the problem of determining when f_{χ_j} becomes isomorphic under the condition that a certain associated affine toric variety is normal.

In § 1 and § 2, we define the system M_α and the natural morphism f_{χ_j}, and give a necessary condition (Theorem 2.3) for the morphism f_{χ_j} to be an isomorphism. In § 3, we introduce an assumption, which we call the normality and keep throughout this paper. In § 4, § 5, and § 6, we define an ideal $B(\chi_j)$ of the b-functions for the morphism f_{χ_j}, and obtain a sufficient condition in terms of the b-functions (Corollary 5.4) for the morphism f_{χ_j} to be isomorphic. The ideal $B(\chi_j)$ turns out to be singly generated by a certain polynomial (Theorem 6.4). In § 7, some example are given.

The author would like to thank Professors Ryoshi Hotta and Masa-Nori Ishida for helpful conversation.

1. Generalized hypergeometric systems. First of all, we recall the definition of generalized hypergeometric systems following Gelfand et al. (cf. [GGZ]). Suppose we are given N integral vectors $\chi_j=(\chi_{1j}, \dots, \chi_{nj}) \in \boldsymbol{Z}^n$ $(j=1, \dots, N)$ satisfying two conditions.

(1) The vectors $\chi_1, \dots, \chi_N$ generate the lattice $\boldsymbol{Z}^n$.

---

* Partly supported by the Grants-in-Aid for Encouragement of Young Scientists, The Ministry of Education, Science and Culture, Japan.

1991 *Mathematics Subject Classification*. Primary 33C70; Secondary 14M25, 16W50, 39B32.