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QUANTUM MULTILINEAR ALGEBRA

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Abstract. We construct a quantized version of the theory of multilinear algebra, based on Jimbo's solution of Yang-Baxter equation of type $A_{N-1}^{(1)}$. Using this, we discuss the polynomial representations of quantum general linear groups.

CONTENTS

Introduction. Quantum groups are mathematical objects which arose from the study of the quantum inverse scattering method, especially the Yang-Baxter equation. They are very remarkable Hopf algebras and can be considered as q-analogues of Kac-Moody enveloping algebras or of coordinate rings of Lie groups. Not only have they added new aspects to representation theory, but also they have brought to *non-commutative geometry* a remarkable progress, i.e. the discovery of many new examples such as quantum linear algebraic groups, quantum spheres and so on.

In this article, we study quantum analogues of some linear-algebraic objects such as matrices, symmetric and alternating tensors, and determinants. We construct these from Jimbo's solution of Yang-Baxter (YB) equation of type $A_{N-1}^{(1)}$ and investigate their structure via the notion which we call *Yang-Baxter bialgebras*. As applications, we give realizations and free bases of Weyl modules $K_\lambda V$ and their dual modules (Schur modules) of quantum general linear groups $GL_q(N)$, and give a criterion for the irreducibility of $K_\lambda V$. We also give an analogue of the straightening formula for quantum matric bialgebras. We would like to emphasize that these objects are defined over any commutative ring R and any unit element $q \in R^\times$ and are compatible with extensions

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