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OSCILLATIONS OF VOLTERRA INTEGRAL EQUATIONS WITH DELAY

GEORGE KARAKOSTAS, I. P. STAVROULAKIS AND YUMEI WU

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Abstract. The oscillatory behavior of the solutions of a Volterra type equation with delay is investigated. Sufficient conditions on the kernel are given which guarantee that the oscillatory character of the forcing term is inherited by the solutions.

1. Introduction and preliminaries. In this paper we investigate when the oscillatory character of the forcing term f of the Volterra integral equation with delays of the form

$$x(t)=f(t)-\int_0^t K(t, s, x_s)ds\,, \qquad t\geq 0 \tag{1.1}$$

is inherited by the solutions.

As we can see from [2], [6], [27] and the references cited therein, the equations of the type (1.1) arise, for example, in certain applications to impulsive theory. It has also been a very interesting subject to study how the behavior (e.g., boundedness, convergence, periodicity, asymptotic periodicity, slow (or almost slow) varyingness) of the forcing term f produces the same property of the solutions of a Volterra integral equation under certain conditions on the kernel K (cf. [8], [9], [12], [13], [15], [16], [17], [18], [19]). Therefore it is natural to investigate how oscillation of the forcing term f can be inherited by the solutions of (1.1). Our aim here is to establish conditions on the nonlinear (in general) kernel K under which if the function f is oscillatory, strongly, quickly, moderately or slowly oscillatory, then every solution of (1.1) is oscillatory, strongly, quickly, moderately or slowly oscillatory, respectively.

Before giving the definitions of various types of oscillations mentioned above, we have to present some preliminaries needed in the sequel.

Let $C:=C([-r, 0], \boldsymbol{R})$ denote the Banach space of all continuous functions mapping the interval $[-r, 0]$ into $\boldsymbol{R}$ endowed with the sup-norm $\|\cdot\|$. For any function $x: \boldsymbol{R}\mapsto \boldsymbol{R}$ and $t\in \boldsymbol{R}^+$, we define x_t by $x_t(\theta):=x(t+\theta)$, $\theta\in[-r, 0]$.

The following assumptions will be used throughout this chapter without any further mention.

(A_1) $f: \boldsymbol{R}^+\mapsto \boldsymbol{R}$ is continuous.

(A_2) K maps the set $\{(t, s): 0\leq s\leq t, t\geq 0\}\times C$ into $\boldsymbol{R}$.

(A_3) The function $\varphi\mapsto K(t, s, \varphi)$ is continuous, and maps bounded sets into

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