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A DIFFERENTIAL GEOMETRIC PROPERTY OF BIG LINE BUNDLES

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Abstract. A holomorphic line bundle over a compact complex manifold is shown to be big if it has a singular Hermitian metric whose curvature current is smooth on the complement of some proper analytic subset, strictly positive on some tubular neighborhood of the analytic subset, and satisfies a condition on its integral. In particular, we obtain a sufficient condition for a compact complex manifold to be a Moishezon space.

1. Introduction. In this paper, we consider sufficient conditions for a singular Hermitian line bundle (L, h) over a compact complex space X to be big and, consequently, for X to be a Moishezon space. A holomorphic line bundle L is said to be big if $\dim \Phi_{|L^{\otimes\nu}|}(X) = \dim X$ for some $\nu \in \boldsymbol{N}$, where $\Phi_{|L^{\otimes\nu}|}$ is the meromorphic map to some $\boldsymbol{P}^N$ by means of the global sections of $L^{\otimes\nu}$. A reduced and irreducible compact complex space X is said to be a Moishezon space if the transcendence degree of the meromorphic function field of X over a complex number filed $\boldsymbol{C}$ is equal to the dimension of X. By Moishezon [Mo], X is a Moishezon space if and only if there exists a bimeromorphic holomorphic map from a projective manifold to X. Our problem arose from an attempt to generalize the following theorem due to Kodaira [Ko]: A compact complex manifold is projective algebraic if and only if there exists a positive line bundle on it. There are several works in this direction; [De], [G-R], [Ri], [Si 1], [Si 2], [Ji 1], [Ji 2], [J-S] and so on. The former five works are related to *smooth* Hermitian metrics, and their theorems are motivated by the conjecture of Grauert and Riemenschneider: A compact complex manifold admits a smooth Hermitian holomorphic line boundle whose curvature form is positive definite on a dense subset of it, then it is Moishezon. However, it is not enough to characterize Moishezon spaces by smooth metrics as is mentioned below. Let X be a non-projective Moishezon manifold. Then there exists a proper modification $\pi: \tilde{X} \to X$ from a projective manifold. By Kodaira, $\tilde{X}$ carries a smooth integral Kähler form $\tilde{\omega}$. Then the push-forward $\pi_*\tilde{\omega}$ is an integral Kähler *current* which is smooth on the complement of some proper analytic subset of X. However, X does not have a smooth Kähler *form* (by [Mo], Kähler and Moishezon imply projectivity). On the other hand, [Ji 1], [Ji 2], [J-S] are related to *singular* Hermitian metrics. Ji and Shiffman [J-S] proved the conjecture of Shiffman: A compact complex manifold

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