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POSITIVE SOLUTIONS WITH WEAK ISOLATED SINGULARITIES TO SOME SEMILINEAR ELLIPTIC EQUATIONS

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Abstract. We are concerned with the existence of positive solutions with prescribed weak isolated singularities to some semilinear elliptic equations. The existence property differs with the behavior of the nonlinear term. Under the positivity assumption and a growth condition on the nonlinear term, we obtain not only solutions with a finite number of singularities but also those with infinitely many singularities. We show also that, for some nonlinear terms which changes sign, there is no solution with prescribed singular behavior.

1. Introduction. In this paper we are concerned with the problem of finding solutions with isolated singularities to some semilinear elliptic partial differential equations. Choosing a finite set of points $\{a_j\}_{j=1}^m$ or a sequence $\{a_j\}_{j=1}^\infty$ without accumulation points in $\boldsymbol{R}^N$ and a bounded set of positive numbers $\{\kappa_j\}_{j=1}^m$ or $\{\kappa_j\}_{j=1}^\infty$ arbitrarily, we consider the following problems:

$$(\mathrm{P}_m)\qquad \begin{cases} -\Delta u+f(u)=0 \quad \text{and} \quad u>0 \quad \text{in } \boldsymbol{R}^N\setminus\{a_j\}_{j=1}^m\,, \\ u(x)\sim\kappa_j E(x-a_j) \quad \text{as } x\to a_j \quad \text{for } j=1,2,\dots,m\,, \\ u(x)\to 0 \quad \text{as } |x|\to+\infty\,, \end{cases}$$

and

$$(\mathrm{P}_\infty)\qquad \begin{cases} -\Delta u+f(u)=0 \quad \text{and} \quad u>0 \quad \text{in } \boldsymbol{R}^N\setminus\{a_j\}_{j=1}^\infty\,, \\ u(x)\sim\kappa_j E(x-a_j) \quad \text{as } x\to a_j \quad \text{for } j=1,2,\dots\,. \end{cases}$$

Here $\Delta:=\sum_{i=1}^N(\partial/\partial x_i)^2$ is the Laplacian in $\boldsymbol{R}^N$ with $N\geq 2$ and E is the fundamental solution for $-\Delta$ in $\boldsymbol{R}^N$, that is,

$$(1.1)\qquad E(x)=E(|x|):=\begin{cases} \dfrac{1}{(N-2)N\omega_N}\dfrac{1}{|x|^{N-2}} & \text{if} \quad N\geq 3 \\ \dfrac{1}{2\pi}\log\dfrac{1}{|x|} & \text{if} \quad N=2 \end{cases} \qquad \text{for } x\in\boldsymbol{R}^N\setminus\{0\}\,,$$

where ω_N denotes the volume of the unit ball in $\boldsymbol{R}^N$. We assume that $f:[0,+\infty)\to\boldsymbol{R}$ is (locally) Lipschitz continuous and $f(0)=0$. We call the number κ_j the *intensity* of

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