Tôhoku Math. J.
47 (1995), 31–54

MINIMAL SURFACES IN R^3 WITH DIHEDRAL SYMMETRY

WAYNE ROSSMAN

(Received August 31, 1993, revised February 8, 1994)

Abstract. We construct new examples of immersed minimal surfaces with catenoid ends and finite total curvature, of both genus zero and higher genus. In the genus zero case, we classify all such surfaces with at most $2n+1$ ends, and with symmetry group the natural Z_2 extension of the dihedral group D_n.

The surfaces are constructed by proving existence of the conjugate surfaces. We extend this method to cases where the conjugate surface of the fundamental piece is noncompact and is not a graph over a convex plane domain.

1. Introduction. Recently, new examples of immersed minimal surfaces of finite total curvature with catenoid ends have been found. Among these examples are: the genus-zero Jorge-Meeks n-oid with symmetry group $D_n \times Z_2$ [JoMe], the genus-one n-oid with symmetry group $D_n \times Z_2$ [BeRo], the genus-zero Platonoids with symmetry groups isomorphic to the symmetry groups of the Platonic solids [Xu], [Kat], [UmYa], and the higher genus Platonoids with Platonic symmetry groups [BeRo]. (See Figures 1(1)–(4), 2(1).) By $D_n \times Z_2$, we mean the natural Z_2 extension into $O(3)$ of the dihedral group $D_n \subset SO(3)$.

In this present work we find more examples with symmetry group $D_n \times Z_2$ (see Figures 2(2)–(4), 3(1)–(3)), of both genus zero and higher genus. Then, in the genus zero case, we classify all such surfaces that have at most $2n+1$ ends.

To prove existence of these surfaces we use the conjugate surface construction, by an approach similar to that of [BeRo]. Generally speaking, the conjugate surface construction seems to require a high degree of symmetry of the surface. In fact, all of the known techniques for creating examples of minimal surfaces seem to benefit from symmetry assumptions.

The examples we construct here are less symmetric than the examples mentioned in the first paragraph, in the sense that their fundamental pieces have higher total Gaussian curvature. It is therefore harder to prove existence of the conjugate surfaces to these fundamental pieces. Hence the constructions we need are more delicate than those in [BeRo]. For less symmetric surfaces, the conjugates may no longer lie over convex plane domains, thus making Nitsche's theorem no longer applicable. In fact, they may not even be graphs, or may not even be embedded.

1991 *Mathematics Subject Classification*. Primary 53A10; Secondary 49Q05, 53C42.

This research was supported by a fellowship from the Japan Society for the Promotion of Science.