Tôhoku Math. J.
47 (1995), 15–29

A DIFFERENTIABLE SPHERE THEOREM BY CURVATURE PINCHING II

Dedicated to Professor Shoshichi Kobayashi on his sixtieth birthday

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(Received May 31, 1993, revised August 19, 1994)

Abstract. We give a new diffeotopy theorem on the standard sphere, and an estimate for some geometric invariants concerning positively curved Riemannian manifold. By using these results we prove that a complete, simply connected and 0.654-pinched Riemannian manifold is diffeomorphic to the standard sphere.

Introduction. Let (M^n, g) be a complete, simply connected and δ-pinched Riemannian n-manifold. In this paper we prove that if $\delta=0.654$, then M is diffeomorphic to the standard sphere S^n.

For a $\delta(>1/4)$-pinched Riemannian n-manifold, an orientation preserving diffeomorphism f of S^{n-1} is naturally defined, and is used in the proof of the differentiable sphere theorem [3, 4]. In fact, if there exists a diffeotopy from f to an isometry f_1 of S^{n-1}, then M is diffeomorphic to the standard sphere. In order to find the minimum of such δ's it is important to construct a diffeotopy in as many different ways as possible. In this paper, we propose a new construction of a diffeotopy. The statement of our diffeotopy theorem and the construction of diffeotopy in it are fairly simple in comparison with these in [4]. Furthermore, by giving new estimates concerning f and its differential df we prove the differentiable sphere theorem above. In this paper we use the same notation as in [4, §2–§6].

The author would like to thank the referees for careful reading of the previous versions of this paper and for valuable suggestions for improvements.

1. $\delta(>1/4)$-pinched Riemannian manifolds. Let (M^n, g) be a complete, simply connected and $\delta(>1/4)$-pinched Riemannian n-manifold, i.e., the sectional curvature K of M satisfies $\delta\leq K\leq 1$ everywhere. We denote by D the Levi-Civita connection induced by the Riemannian metric g. First, we review the definitions of the diffeomorphism f, mentioned in the Introduction, and the differentiable map $\alpha: S^{n-1}\ni x\mapsto\alpha_x\in SO(n, \boldsymbol{R})$, which is regarded as an approximation of df, and related results in (A) and (B) below (cf. [4]). Let S^{n-1} be the standard sphere with sectional curvature 1, i.e., $S^{n-1}=S^{n-1}(1)$. We denote by $d_s(x, y)$ the distance between x and y

1991 *Mathematics Subject Classification*. Primary 53C20.