Tôhoku Math. J.
47 (1995), 613–614

ADDENDUM: ON $\boldsymbol{Q}$-STRUCTURES OF QUASISYMMETRIC DOMAINS

(Tôhoku Math. J. 47 (1995), 357–390)

ICHIRO SATAKE

(Received August 22, 1995)

In A3, Theorem 3 (p. 389), the condition (β) was unnecessary and could be dropped. Actually, one can prove the following

LEMMA A. *Let $\mathscr{S}_I$ be a quasisymmetric domain with a $\boldsymbol{Q}$-structure (in the sense of the text) and suppose that $\mathscr{S}_I$ is symmetric and $\mathfrak{G}=\mathrm{Lie\,Aut}\,\mathscr{S}_I$ has a $\boldsymbol{Q}$-structure extending the given $\boldsymbol{Q}$-structure of $\mathfrak{G}_{\mathrm{Aff}}$. Then, for any Cartan involution θ of $\mathfrak{G}$ at $(ie, 0)\in\mathscr{S}_I$ with e semirational, the map $\theta|U$ is $\boldsymbol{Q}$-rational.*

PROOF. By the assumption $X_0=(1_U, (1/2)1_V)$ $(\in\mathfrak{G}_{\mathrm{Aff}})$ is $\boldsymbol{Q}$-rational, so that the subspaces $\mathfrak{G}_{\nu/2}$ $(\nu=0, \pm 1, \pm 2)$ are all defined over $\boldsymbol{Q}$. The Cartan involution θ induces a linear isomorphism $\mathfrak{G}_1 \rightleftarrows \mathfrak{G}_{-1}=U$ and one has $(\mathrm{ad}\,e)^2\mathfrak{G}_1\subset\mathfrak{G}_{-1}$; moreover, $(\mathrm{ad}\,e)^2$ is $\boldsymbol{Q}$-rational for e semirational. Hence our assertion follows from the relation

$$\theta|U=2((\mathrm{ad}\,e)^2|\mathfrak{G}_1)^{-1}.$$

(Cf. [S7, (10), (11b), and (12)] in the References of the text.) q.e.d.

Therefore Theorem 3 can be stated in the following form.

THEOREM 3A. *Let $\mathscr{S}_I$ be a quasisymmetric domain with a $\boldsymbol{Q}$-structure, and suppose that $\mathscr{S}_I$ is symmetric. Then $\mathfrak{G}=\mathrm{Lie\,Aut}\,\mathscr{S}_I$ has a unique $\boldsymbol{Q}$-structure extending the given $\boldsymbol{Q}$-structure of $\mathfrak{G}_{\mathrm{Aff}}$. A Cartan involution θ of $\mathfrak{G}$ at $(ie, 0)\in\mathscr{S}_I$ is $\boldsymbol{Q}$-rational if and only if e is semirational and compatible with I. (In particular, there exists always a $\boldsymbol{Q}$-rational Cartan involution of $\mathfrak{G}$.)*

The uniqueness in the first assertion and the second assertion on Cartan involutions are shown in the same way as in the proof of Theorem 3 by just replacing condition (β) by Lemma A above. To prove the existence in the first assertion, take a semirational element $e\in\mathscr{C}$, compatible with I, and let θ be the Cartan involution of $\mathfrak{G}$ at $(ie, 0)$. Then θ induces a $\boldsymbol{Q}$-rational Cartan involution of $\mathfrak{G}_0=\mathfrak{g}_1+\mathfrak{k}_2$. Define the $\boldsymbol{Q}$-structure of the subspaces $\mathfrak{G}_{\nu/2}$ $(\nu=1, 2)$ so that the maps $\theta: \mathfrak{G}_{-\nu/2}\to\mathfrak{G}_{\nu/2}$ are $\boldsymbol{Q}$-rational. Then, by virtue of (88), (89), (90), and the Lemma (in the text), it is easy to see that the Lie product $(x, y)\to[x, y]$ is $\boldsymbol{Q}$-rational. Thus one obtains a $\boldsymbol{Q}$-structure of $\mathfrak{G}$ extending the given $\boldsymbol{Q}$-structure of $\mathfrak{G}_{\mathrm{Aff}}$.