Tôhoku Math. J.
47 (1995), 595–600

LOGARITHMIC DIVERGENCE OF HEAT KERNELS ON SOME QUANTUM SPACES

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(Received June 28, 1994, revised February 27, 1995)

Abstract. Asymptotic behaviour of the heat kernels on some explicitly known quantum spaces are studied. Then the heat kernels are shown to be logarithmically divergent. These results suggest to us that the "dimensions" of these quantum spaces would not be zero but less than one so that these quantum spaces look almost like "discrete spaces".

Introduction. In spectral geometry, there is a famous asymptotic formula of McKean and Singer [4] which relates the spectrum of the Laplacian and differential geometrical data (volume, the integration of the scalar curvature etc.) of a compact closed manifold. Namely, let $\{\lambda_1, \lambda_2, \dots\}$ be the spectrum of the Laplacian Δ (including the multiplicity) of a given compact closed manifold M of dimension n. Then the asymptotic behaviour of the heat kernel

$$H(t) := \mathrm{Trace}(e^{-t\Delta}) = \sum_{j=1}^{\infty} e^{-\lambda_j t}$$

for $t \downarrow 0$ is given by

$$\left(\frac{1}{4\pi t}\right)^{n/2} \{a_0 + a_1 t + a_2 t^2 + \cdots\},$$

where $a_0 = \mathrm{Volume}(M)$ is the volume of the manifold M and a_1 is the amount given in terms of the scalar curvature $\kappa(x)$ of the manifold M by

$$a_1 = \int_M \kappa(x) dv(x).$$

The higher degree terms $a_2, a_3 \dots$ are also expressed in terms of differential geometrical data. Then this asymptotic exapansion formula tells us in particular that the dimension of a given manifold M is determined by the behaviour of the spectrum of the Laplacian on M.

This paper is devoted to the study of the asymptotic behaviour of the heat kernels associated with the quantum group $SU_q(2)$ and the quantum two-sphere $S_q^2(c, d)$ of Podleŝ. The motivation of our study comes from the fact that in the case of quantum

1991 *Mathematics Subject Classification*. Primary 58G25; Secondary 58B30, 33D80, 17B37.