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WAVELET TRANSFORMS IN EUCLIDEAN SPACES —THEIR RELATION WITH WAVE FRONT SETS AND BESOV, TRIEBEL-LIZORKIN SPACES—

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Abstract. We define a class of wavelet transforms as a continuous and microlocal version of the Littlewood-Paley decompositions. Hörmander's wave front sets as well as the Besov and Triebel-Lizorkin spaces may be characterized in terms of our wavelet transforms.

Introduction. We define a class of wavelet transforms as a continuous and micro-local version of the Littlewood-Paley decompositions. Hörmander's wave front sets as well as the Besov and Triebel-Lizorkin spaces may be characterized in terms of our wavelet transforms. We remark that the components of our decompositions are not linearly independent but can be treated as if they were. This paper consists of two parts. The first part treats the comparison between the wave front sets defined by our wavelet transforms and Hörmander's wave front sets. The second part gives the characterization of the Besov and Triebel-Lizorkin spaces by using our wavelet transforms.

First, we define our wavelet transforms as follows:

DEFINITION 1. Suppose that a function $\psi(x)$ (called a wavelet) has the following properties: $\psi(x)\in\mathscr{S}(\boldsymbol{R}^n)$, $\hat{\psi}(\xi)\in C_0^\infty(\boldsymbol{R}^n)$ and $\hat{\psi}(\xi)\geqq 0$. Let $\Omega=\operatorname{supp}\hat{\psi}(\xi)$ be in a neighbourhood of $(0,\dots,0,1)$. When $n=1$, $\Omega\subset(0,\infty)$, while when $n\geqq 2$, Ω is connected, does not contain the origin 0 and $\psi(x)=\psi(rx)$ for any $r\in SO(n)$ satisfying $r(0,\dots,0,1)=(0,\dots,0,1)$. Let r_ξ be any rotation which sends $\xi/|\xi|$ to $(0,\dots,0,1)$. Then our wavelet transform is defined as follows: for $f(t)\in\mathscr{S}'(\boldsymbol{R}^n)$, $(x,\xi)\in\boldsymbol{R}^{2n}$,

$$W_\psi f(x,\xi)=\begin{cases}\int_{\boldsymbol{R}} f(t)|\xi|^{1/2}\overline{\psi(\xi(t-x))}\,dt\,, & \text{if}\quad n=1\,,\\ \int_{\boldsymbol{R}^n} f(t)|\xi|^{n/2}\overline{\psi(|\xi|r_\xi(t-x))}\,dt\,, & \text{if}\quad n\geqq 2\,.\end{cases}$$

Here $\mathscr{S}(\boldsymbol{R}^n)$ stands for the Schwartz class and $C_0^\infty(\boldsymbol{R}^n)$ consists of functions which are smooth and compactly supported.

REMARK 1. $W_\psi f(x,\xi)$ is rewritten as follows:

$$\int_{\boldsymbol{R}^n}\hat{f}(\tau)\cdot|\xi|^{-n/2}\hat{\psi}(|\xi|^{-1}r_\xi\tau)\cdot e^{it x}d\tau\,.$$

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