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CONSTANT MEAN CURVATURE HYPERSURFACES IN NONCOMPACT SYMMETRIC SPACES

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Abstract. Here, we compute the mean curvature of the geodesic sphere at any point in some symmetric spaces and determine the lower bound of the mean curvature of a closed hypersurface of constant mean curvature in it. With the Hessian Comparison Theorem, we also show that there is a lower bound for the mean curvature of any closed hypersurface of constant mean curvature in a manifold with a pole satisfying a curvature condition.

1. Introduction. In this article, we study closed hypersurfaces of constant mean curvature in noncompact symmetric spaces or, more generally, the product of such spaces with a Euclidean space. These closed hypersurfaces of constant mean curvature are called soap bubbles in [HH89] and we refer the readers to this paper as well as [Kap90], [Kap91] and the references there for a discussion of the historical as well as mathematical background of these hypersurfaces. Our main theorem in this direction is the determination of a lower bound of the mean curvature of these hypersurfaces in terms of $\Lambda(M)$, defined as follows. Let M be such a space and let p be any point in M. For $v\in T_pM$, define a symmetric linear map $K_v\colon T_pM\to T_pM$ by

$$K_v(X)=R(X, v)v\,, \qquad \text{for} \quad X\in T_pM\,.$$

We let

$$\Lambda(M)=\max\left\{\sum_{i=1}^{n} c_1(v)\colon v\in T_p(M) \text{ and } \|v\|=1\right\}$$

where $\{c_1(v)^2, \dots, c_n(v)^2\}$ are all the eigenvalues of K_v. Throughout this paper, we assume that all the c_i's are nonnegative without loss of generality. This lower bound should be compared with an earlier result in the same direction in [Hsi92]. While Hsiang's result is in terms of roots, we shall show that the bound we obtain here is at least as big as that of [Hsi92]; whether or not they are equal is unclear at this point.

With the Hessian Comparison Theorem, we also prove that there is a lower bound for the mean curvature of any closed hypersurface of constant mean curvature in a manifold with a pole when its radial curvature is $\leq -c^2$ for some nonzero constant c.

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